Relief Valve Sizing for the FLARE Materials Test Station Cryostat

The pressure relief devices for the FLARE Materials Test Station Cryostat were sized according to the Compressed Gas Association's CGA S-1.3—1995 document. This document is entitled, "Pressure Relief Device Standards Part 3—Stationary Storage Containers for Compressed Gases." In section 4.1.1 it states, "…each container shall be provided with a primary system of one or more pressure relief devices and a secondary system of one or more pressure relief valves or rupture disks or buckling pin devices."

This vessel (PPD ID# 10100) is equipped with two pressure relief valves (PSV-210-Ar and RD-302-Ar). The basic vessel geometry is shown in Figure 1. The relief valve is set at the vessel MAWP of 35 psig while the rupture disk is set at 55 psig which is slightly less than 150% of MAWP.

First the fire condition is considered as it is more difficult to relieve. To begin the calculation, an estimate of the relief capacity required is computed. This number is then corrected for pressure drop and temperature rise in the line that leads to the reliefs if required. In section 5.3.3 the following equation is used to calculate the minimum required flow capacity

$$Q_a = FG_iUA^{0.82}$$

where:

U = Overall heat transfer coefficient to the liquid, $\frac{Btu}{hr \cdot ft^2 \cdot {}^oF}$.

F = Correction factor for pressure drop and temperature rise in line to relief valve.

A = Average surface area of the inner and outer vessels, 25.90 ft^2 (see Figure 2 for key dimensions).

G_i = Gas factor for insulated containers.

Q_a = Flow capacity required at applicable flow rating pressure and 60 ^{o}F in cubic feet per minute of free air.

First the overall heat transfer coefficient to the liquid must be computed. For the fire condition it was assumed that the outer vessel is exposed to an environment that is at 1200 ^{o}F (922 K) and the vacuum space between the inner and outer vessel has been filled with air at atmospheric pressure (air has a higher thermal conductivity than argon). The inner vessel wall will be at the saturation temperature of liquid argon at the flow rating pressure. The super insulation around the inner vessel is ignored because it may deteriorate in a fire. The relief valve is set at 35 psig. For the fire condition it must be ensured that the pressure does not exceed 121% MAWP. Thus the flow rating pressure is 1.21(35+15) -15, or 45.5 psig. The saturation temperature of liquid argon at 45.5 psig is 185.8 ^{o}R (103.2 K).

Several heat transfer mechanisms are considered for the fire condition. Two separate heat transfer paths are modeled. The first path involves convection and radiation from the environment to the vertical sidewalls of the cryostat, conduction thru these sidewalls, convection and radiation thru the annular vacuum space while filled with air, and conduction thru the inner vessel sidewall into the liquid argon. The second path considers convection and radiation to the thick top flange of the cryostat, conduction thru this flange, and radiation from this flange to the liquid argon. Convection from the top flange to the liquid argon is not considered because the venting gas will not flow in a manner that transfers heat to the surface of the cryogen. The vented gas will intercept some of the heat arriving from the top flange before it reaches the liquid surface. This reduction in heat input is ignored due to the difficulty of calculating heat transfer from a multi-dimensional gas flow. Its neglection is a conservative assumption. The two heat transfer paths are only coupled in that they both transfer heat into the liquid argon. Heat transfer to the bottom of the cryostat was considered negligible because the bottom of the cryostat is flush with the concrete floor and will not be exposed to fire. All heat transfer equations were solved simultaneously in EES (Engineering Equation Solver) which provided temperature and pressure dependent fluid properties and temperature dependent solid properties.

First the calculations related to path 1 are described. Figure 2 helps relate the equations to the cryostat. The details of the EES computation file are available in the appendix. The heat rates given in watts are the exact solution given by EES. The equations listed in this document have rounded values that when computed won't match the listed heat rate exactly.

Radiation heat transfer from the environment to the outer vessel vertical walls was modeled as a small convex object in a large cavity (Equation 13.27 from Incropera and Dewitt) where

$$q_{1-2rad} = \sigma A_2 \varepsilon_2 \left(T_1^4 - T_2^4\right) = \frac{5.67x10^{-8}W}{m^2 \cdot K^4}\left(2.1682m^2\right)(0.7)\left[(922K)^4 - (867.72K)^4\right] = 13402W .$$

Convective heat transfer to the outer vessel walls was modeled as free convection on a vertical flat plate combining equations 9.24, 9.25, and 9.26, from Incropera and Dewitt

$$q_{1-2conv} = \left\{0.825 + \frac{0.387\left(\frac{g\beta\left(T_1 - T_2\right)L^3}{\alpha_{air}\nu_{air}}\right)^{1/6}}{\left[1+\left(\frac{0.492}{\Pr}\right)^{9/16}\right]^{8/27}}\right\}^2 \left(\frac{k_{air}}{L_{ext}}\right) A_s \left(T_1 - T_2\right)$$

$$q_{1-2conv} = \left\{0.825 + \frac{0.387\left(\frac{9.81\frac{m}{s^2}\frac{1}{\frac{(922+867.72)}{2}K}(922-867.72)K(1.118m)^3}{\left(0.0001404\frac{m^2}{\sec}\right)\left(0.00009898\frac{m^2}{\sec}\right)}\right)^{1/6}}{\left[1+\left(\frac{0.492}{0.7049}\right)^{9/16}\right]^{8/27}}\right\}^2 \left(\frac{0.06202\frac{W}{m\cdot K}}{1.118m}\right)2.168m^2(922-867.72)K = 342W$$

Conduction thru the thin stainless steel vacuum jacket is included in the model and the thermal resistance it presents is negligible. Conduction is computed from Incropera and Dewitt's equation 3.27 which gives the heat transfer rate for radial conduction in a cylinder

$$q_{2-3cond} = \frac{2\pi L_{ext} k_{ss}\left(T_2 - T_3\right)}{\ln\left(\frac{r_3}{r_2}\right)} = \frac{2\pi(1.118m)\frac{23.544W}{m\cdot K}(867.718-867.126)K}{\ln\left(\frac{0.30877m}{0.30658m}\right)} = 13744W .$$

Radiation exchange between the vacuum jacket and the inner vessel was computed using equation 13.25 from Incropera and Dewitt which applies to concentric cylinders.

$$q_{3-5rad} = -\frac{\sigma A_5\left(T_5^4 - T_3^4\right)}{\frac{1}{\varepsilon_5} + \frac{1-\varepsilon_3}{\varepsilon_3}\left(\frac{r_5}{r_3}\right)} = \frac{\frac{5.67x10^{-8}W}{m^2 \cdot K^4}\left(1.987m^2\right)\left(867.126^4 - 105.028^4\right)K^4}{\frac{1}{0.1} + \frac{1-0.7}{0.7}\left(\frac{0.2830m}{0.3066m}\right)} = 6126W$$

The convective heat transfer rate across the thin layer of air in the annular space was determined using equation 4.101 from A. F. Mills which gives the three correlations for the Nusselt number (shown below) for large aspect ratio enclosures with heated and cooled walls and recommends using the largest Nusselt number of the three, which for this case is Nu_2.

$$Nu_1 = 0.0605 Ra_L^{1/3}, Nu_2 = \left\{1 + \left[\frac{0.104 Ra_L^{0.293}}{1+\left(\frac{6310}{Ra_L}\right)^{1.36}}\right]^3\right\}^{1/3}, Nu_3 = 0.242\left(\frac{Ra_L}{\frac{H}{L}}\right)^{0.272}$$

Combining the standard relationships for Nusselt number, Rayleigh number, and the convective heat transfer rate equation yields

$$q_{3-5conv} = \left\{1 + \left[\frac{0.104\left(\frac{g\beta\left(T_5 - T_3\right)L_{annular}{}^3}{\alpha_{air}\nu_{air}}\right)^{0.293}}{1+\left(\frac{6310}{\left(\frac{g\beta\left(T_5 - T_3\right)L_{annular}{}^3}{\alpha_{air}\nu_{air}}\right)}\right)^{1.36}}\right]^3\right\}^{1/3}\left(\frac{k_{air}}{L_{annular}}\right)A_5\left(T_5 - T_3\right)$$

which results in the follow when the numbers are plugged in.

$$q_{3-5conv} = \left\{1 + \left[\frac{0.104\left(\dfrac{9.81\dfrac{m}{s^2}\dfrac{1}{\dfrac{(867.126+105.028)}{2}K}(867.126-105.028)K(0.0236217m)^3}{\left(0.000051899\dfrac{m^2}{\sec}\right)\left(0.000036148\dfrac{m^2}{\sec}\right)}\right)^{0.293}}{\left(1+\left(\dfrac{6310}{\left(\dfrac{9.81\dfrac{m}{s^2}\dfrac{1}{\dfrac{(867.126+105.028)}{2}K}(867.126-105.028)K(0.0236217m)^3}{\left(0.000051899\dfrac{m^2}{\sec}\right)\left(0.000036148\dfrac{m^2}{\sec}\right)}\right)}\right)\right)^{1.36}}\right]^3\right\}^{1/3}\left(\frac{0.03864\dfrac{W}{m\cdot K}}{0.0236217m}\right)1.9869m^2(867.126-105.028)K = 7618W$$

Conduction thru the thin stainless steel inner vessel wall is included in the model and the thermal resistance is negligible. Conduction is computed from Incropera and Dewitt's equation 3.27

$$q_{5-6cond} = \frac{2\pi L_{ext}k_{ss}(T_5 - T_6)}{\ln\left(\dfrac{r_6}{r_5}\right)} = \frac{2\pi(1.1176m)\dfrac{9.3404W}{m\cdot K}(105.0282-103.2336)K}{\ln\left(\dfrac{0.282956m}{0.280543m}\right)} = 13744W$$

The second heat transfer path starts with radiation and convective heat transfer to the top flange of the cryostat. The hardware attached to the top of the flange is ignored. Although the attached hardware increases the surface area of the flange, the contact resistances at the flanged attachment points and the thermal resistance associated with conduction thru the thin walls of the stainless steel tubes that support the flanges greatly limits additional heat input into the flange.

Radiation heat transfer to the top and sides of the flange was modeled as a small convex object in a large cavity (Equation 13.27 from Incropera and Dewitt) where

$$q_{1-11rad} = \sigma A_{11}\varepsilon_{11}\left(T_1^4 - T_{11}^4\right) = \frac{5.67x10^{-8}W}{m^2\cdot K^4}\left(0.5225m^2\right)(0.7)\left[(922K)^4 - (844.236K)^4\right] = 4452W.$$

Convective heat transfer to the top flange is modeled as the upper surface of a cooled plate using equation 9.32 from Incropera and Dewitt which results in the following when the Rayleigh number and Nusslet number are plugged into the convective heat transfer equation.

$$q_{1-11conv} = 0.27\left(\frac{g\beta(T_1 - T_{11})L_{top}^{\ 3}}{\alpha_{air}\nu_{air}}\right)^{1/4}\left(\frac{k_{air}}{L_{top}}\right)A_{11}(T_1 - T_{11})$$

$$q_{1-11conv} = 0.27\left(\frac{9.81\frac{m}{s^2}\frac{1}{\frac{(922+844.236)}{2}K}(922-844.236)K(0.2381m)^3}{\left(0.0001376\frac{m^2}{\sec}\right)\left(0.00009689\frac{m^2}{\sec}\right)}\right)^{1/4}\left(\frac{0.06143\frac{W}{m\cdot K}}{0.2381m}\right)0.5225m^2(922-844.236)K = 86.57W$$

The radiation and convective heat loads are then conducted thru the top flange which was modeled as 1D conduction using the entire cross-sectional area of the flange

$$q_{11-10cond} = \frac{k_{ss}A_{10}(T_{11} - T_{10})}{L_{10}} = \frac{\frac{23.082W}{m\cdot K}(0.3832m^2)(844.236-824.687)K}{0.0381m} = 4538W$$

The heat then radiates from the top flange to the liquid argon. Heat input will cause vapor to be generated which will flow out the relief valve. At the high rate of vapor generation during a fire, a convection cell transferring heat from the underside of the flange to the liquid surface will not form. Instead, vapor leaving the cryostat will remove heat from the flange as it exits. The vapor being relieved intercepts heat before it reaches the liquid argon. Due to the difficulty of modeling heat transfer that results from a three dimensional gas flow, this interception of heat is ignored which is the conservative approach.

Radiation from the top flange to the liquid argon is modeled as exchange between two parallel planes using Incropera and Dewitt equation 13.24 where

$$q_{10-12rad} = \frac{A_{12}\sigma(T_{10}^4 - T_{12}^4)}{\frac{1}{\varepsilon_{10}} + \frac{1}{\varepsilon_{12}} - 1} = \frac{0.2473m^2\frac{5.67x10^{-8}W}{m^2\cdot K^4}(824.687^4 - 103.234^4)K^4}{\frac{1}{0.7} + \frac{1}{1.0} - 1} = 4538W$$

The combined heat load from both paths is 13744 + 4538 = 18282 W. For the CGA calculation this must be converted to an overall heat transfer coefficient to the liquid.

$$h = \frac{q}{A\Delta T} = \frac{18282W}{2.408m^2(922-103.234)K}\times\frac{1}{1W}\frac{1J}{\sec}\times\frac{1Btu}{1055.06J}\times\frac{3600\sec}{hr}\times\frac{1m^2}{10.7639ft^2}\times\frac{1K}{1.8R} = 1.633\frac{Btu}{hr\cdot ft^2\cdot{}^\circ F}$$

To calculate the initial estimate of the relief capacity needed, a gas factor, G_i, must be computed. When the flow rating pressure is less than 40% of the critical pressure ($\frac{60.2\,psia}{705.4\,psia}\cdot 100 = 8.5\%$), the following is used to compute G_i.

$$G_i = \frac{73.4(1660 - T)}{CL}\sqrt{\frac{ZT}{M}}$$

where

L = Latent heat of product at flow rating pressure, 63.33 $\frac{Btu}{lb_m}$ for saturated conditions at 60.2 psia.

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C = Constant for vapor related to ratio of specific heats (k=c_p/c_v) at standard conditions. k = 1.67 for Argon at 60 ^{o}F and 14.696 psia which corresponds to C = 378.

Z = Compressibility factor for saturated vapor at 60.2 psia

$$Z = \frac{Pv}{RT,} \qquad Z = \frac{60.2(0.7531)144}{\frac{1545}{39.948}(185.8)} = 0.909\,.$$

T = Flow rating temperature, 185.8 ^{o}R.

M = Molecular weight of gas, 39.948.

v = specific volume, (saturated vapor at flow rating pressure of 60.2 psia, 0.7531 $\frac{ft^3}{lb_m}$.

G_i is caluculated to be $\frac{73.4(1660-185.8)}{378\cdot 63.33}\sqrt{\frac{0.909\cdot 185.8}{39.948}} = 9.30\,.$

The uncorrected volumetric flow rate was found to be

$$Q_{ae} = 1.0\cdot 9.30\cdot 1.633\cdot 25.90^{0.82} = 219\frac{ft^3}{\min} \text{ of free air}$$

The relief valve is attached to the cryostat thru piping of length less than 2 feet, thus the correction factor F does not have to be calculated according to CGA section 5.1.4

The primary relief is an Anderson Greenwood Type 81 with the F orifice. Anderson Greenwood provides the following sizing formula $A = \frac{V\sqrt{MTZ}}{6.32CKP1}$ where

A = required orifice area, in^2.

V = required capacity, 219 SCFM for free air.

M = molecular weight of gas, 29 for air (The CGA formula converts the required argon mass flow rate to air).

T = relief temperature, 520 ^oR for air at standard conditions.

Z = compressibility factor, 1.0

C = gas constant based on ratio of specific heats, 356 for air

K = nozzle coefficient, 0.816 (derived from manufacturer testing)

P1 = inlet flowing pressure, psia = 1.21 x (35 + 15) -15 + 14.7 = 60.2 psia

$$A = \frac{219\sqrt{29 \cdot 520 \cdot 1.0}}{6.32 \cdot 356 \cdot 0.816 \cdot 60.2} = 0.243 in^2$$

The “F” size Anderson Greenwood relief has an orifice of 0.307 in^2 which is larger than the required 0.243 in^2 and thus the relief valve is adequate for the fire condition.

Because the fire condition includes atmospheric air in the vacuum space, the fire calculation also indicates the relief capacity is adequate for an operational loss of insulating vacuum.

There are three heaters in the cryostat that can provide heat input into the liquid argon. Two of the heaters are 250 W, and the third heater is 1500 W. If operated together, they could provide 2000 W of heat into the liquid argon. This is nine times less than the 18282 W considered for the fire condition. Thus the cryostat is adequately relieved when vapor generation from its internal electrical heats is considered.

The cryostat is filled from FNAL stock room high pressure liquid argon dewars. The reliefs on these 160 liter dewars are set at 350 psig. The flow path from the argon dewars to the cryostat has several restrictions such as valves and filters. Normal filling operation involves cooling down the transfer line by venting the argon just before the cryostat. Once liquid appears at the vent, the flow is then directed into the cryostat. To simplify the calculations, it is assumed that liquid at 350 psig exits the stockroom dewars and enters the cryostat at 35 psig. Once in the warm cryostat the liquid is assumed to completely vaporize and exit as room temperature gas. This is a very conservative calculation because the flashing due to the reduction in pressure from 350 psig to 35 psig will result in a large amount of vapor generation. The amount of vapor generated during this constant enthalpy pressure reduction can be calculated as

$$x = \frac{m_{vapor}}{m_{liquid} + m_{vapor}} = \frac{h_{350\,psig\ saturated\ liquid} - h_{35\,psig\ saturated\ liquid}}{h_{35\,psig\ saturated\ gas} - h_{35\,psig\ saturated\ liquid}} = \frac{-211.8 - (-257.4)}{-107.6 - (-257.4)} = 0.304$$

where the enthalpies are in kJ/kg. Thus, ignoring any heat input into the transfer line, the vapor will be 30% of the total mass flow. The area occupied by the vapor is therefore substantial and will lead to the actual mass flow rate being much smaller than the calculated liquid only flow rate.