LAWRENCE RADIATION LABORATORY - UNIVERSITY OF CALIFORNIA
PHYSICS NOTES
MEMO NO. 786

SUBJECT ELECTRON DIFFUSION AND POSITIVE ION CHARGE RETENTION IN LIQUID-FILLED HIGH-RESOLUTION MULTI-STRIP IONIZATION-MODE CHAMBERS.

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1. Introduction

The purpose of this note is two fold: (1) to estimate the amount of electron diffusion occurring in liquids (especially in liquid argon) and to calculate its effect on the properties of high-resolution multi-strip chambers and (2) to calculate the effect of the slow positive ions on the observed pulse height. High-resolution here indicates a situation where the electrode spacing is comparable to the electron diffusion (typically 30μ).

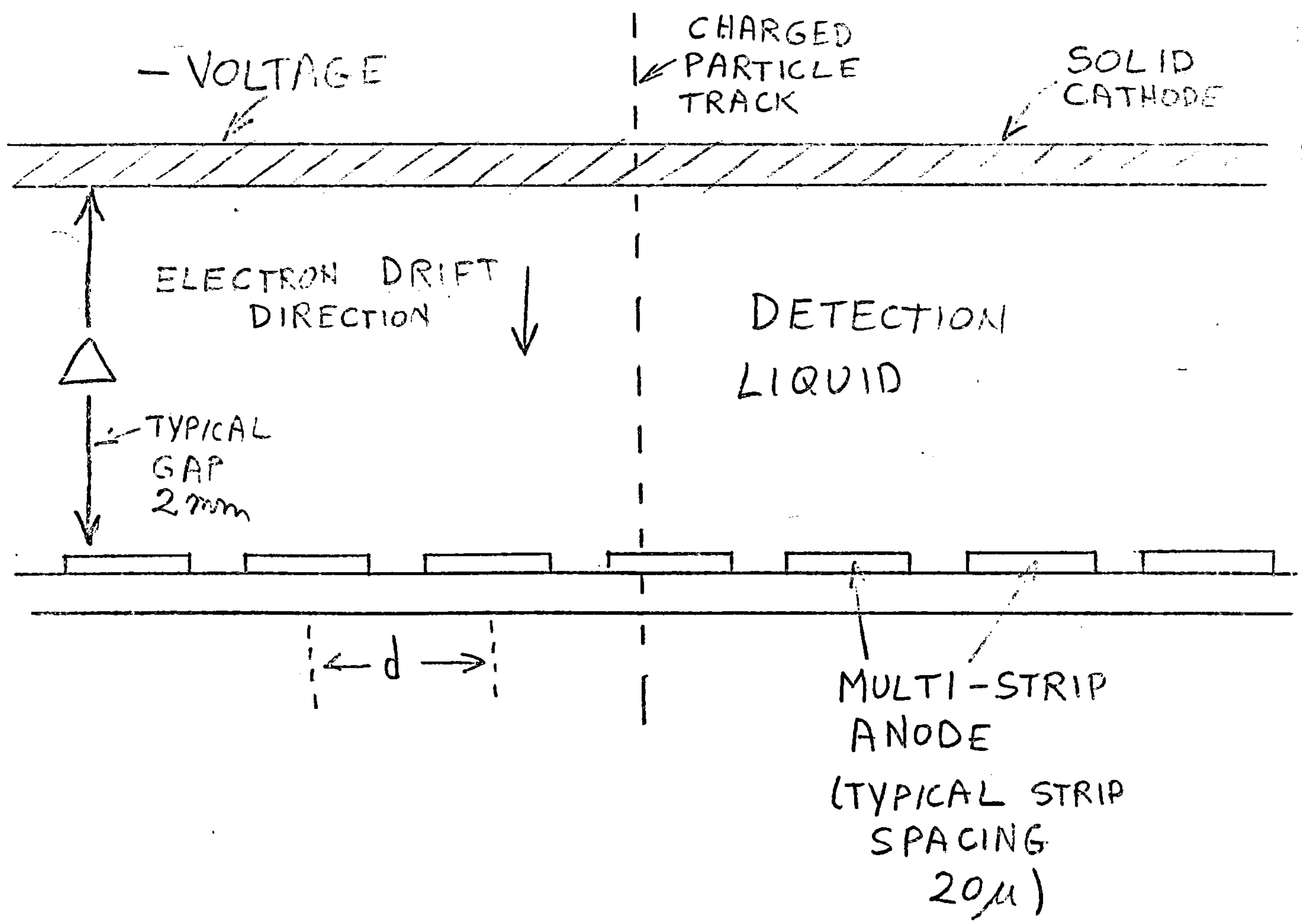

Fig. 1

2. Estimation of Electron Diffusion

Let a = Average electron agitation energy (eV)

e = Electron charge

$\frac{3}{2}$ kT = Average molecular agitation energy (eV)

k = 8.617×10^{-5} eV°K^{-1} Boltzmann constant

D = Diffusion coefficient (cm^2/sec)

$v = \mu E$ = Drift velocity (cm/sec)

E = Electric field (V/cm)

μ = Electron mobility (cm^2/V/sec)

Z = Drift distance parallel to electric field (cm)

$t = Z/v$ = drift time for drift distance Z

σ = rms of the electron diffusion distribution at right angles to electric field (cm)

$$\text{Now } \sigma = \sqrt{2Dt} = \sqrt{\frac{2D}{\mu}\frac{Z}{E}} \tag{1}$$

$$\text{And } a = \frac{3De}{2\mu} \tag{2}$$

For convenience, we also define a diffusion factor $\tilde{D}$

$$\tilde{D} = \frac{\sigma}{\sqrt{Z}} \tag{3}$$

2.1 Room Temperature Hydrocarbons

As a simple example, we consider room temperature hydrocarbons, in which the electron temperature remains essentially at room temperature even for fields as high as 10^5 V/cm. This is evidenced by their nearly constant electron mobility.[1] For these liquids, $a = \frac{3kT}{2} = .038$ eV, and $\frac{2De}{\mu} = \cancel{0.51}\ 0.051$ eV.

At $E = 10^4$ V/cm and $Z = 1$ mm, $\sigma = 7.1 \times 10^{-4}$ cm = 7.1 μm.

2.2 Liquid Argon

In Fig. 2 we plot values of average electron agitation energy $a = \frac{3}{2}\frac{De}{\mu}$ as measured by Warren and Parker[2] in cold (77 - 87° K) Argon gas (where their variable E/p has been multiplied by 6.0×10^5 torr to convert to the electric field E in liquid argon), and values of a calculated for liquid Argon by Lekner[3] based on electron drift velocity measurements. In Fig. 3, we plot values of electron diffusion σ vs. electric field E for Z = 1 mm, based on the curves from Fig. 2 and eqn. (1).

The data points in Figs. 2 and 3 are the only available experimental results, measured using minimum ionizing tracks and a liquid argon multi-strip chamber with Δ = 2.2 mm, d = 20 μm (ref. 4). The diffusion factor $\tilde{D}$ was estimated to be 28 ± 5μ m per $\text{mm}^{1/2}$ at 2.7 kV/cm and essentially the same at 1.4 kV/cm. Using the relationship

$$a = \frac{3}{4}\tilde{D}^2 E \tag{4}$$

the corresponding average electron agitation energies are 0.16 ± 0.05 eV at 2.7 kV/cm and 0.08 ± 0.03 eV at 1.4 kV/cm. The average molecular agitation energy at this temperature (87°K) is 0.011 eV.

In Figs. 2 and 3 we also plot the average agitation energy and diffusion for room temperature liquids.

3. Effects of Electron Diffusion in High-Resolution Multi-Strip Detectors

3.1 Fractional Charge Collected

When a charged particle passes through the detection medium, it leaves behind it a string of electron-ion pairs. If we assume normal incidence (i.e., particle path is orthogonal to detection plane) then the electrons strike the anode electrode with a circular pattern whose intensity has the spread function

Fig. 2

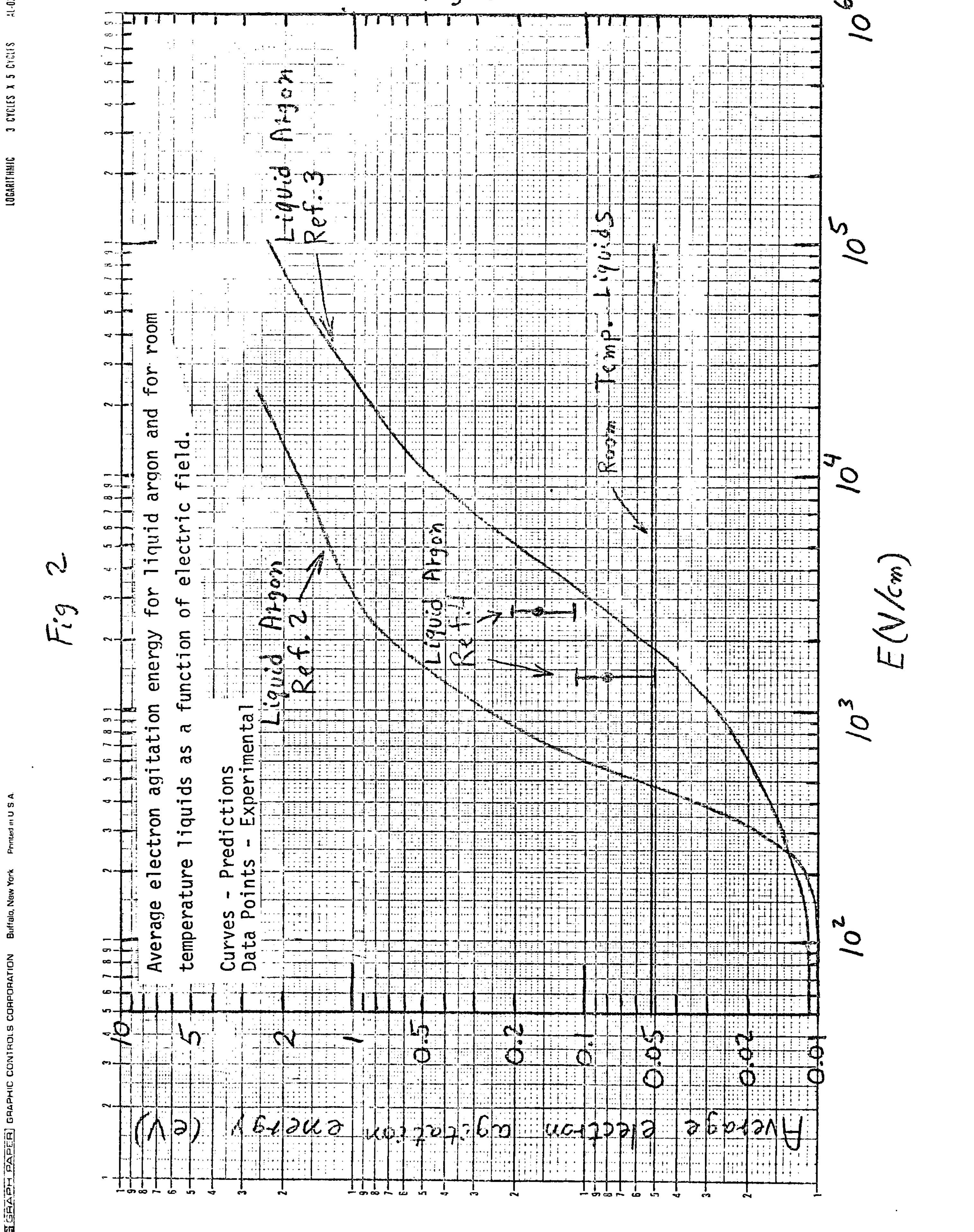
Fig 2
Average electron agitation energy for liquid argon and for room temperature liquids as a function of electric field.
Curves - Predictions
Data Points - Experimental
Liquid Argon Ref. 3
Liquid Argon Ref. 2
Liquid Argon Ref. 1
Room Temp. Liquids
E (V/cm)
Average electron agitation energy (eV)
10
5
2
1
0.5
0.2
0.1
0.05
0.02
0.01
10^2
10^3
10^4
10^5
10^6
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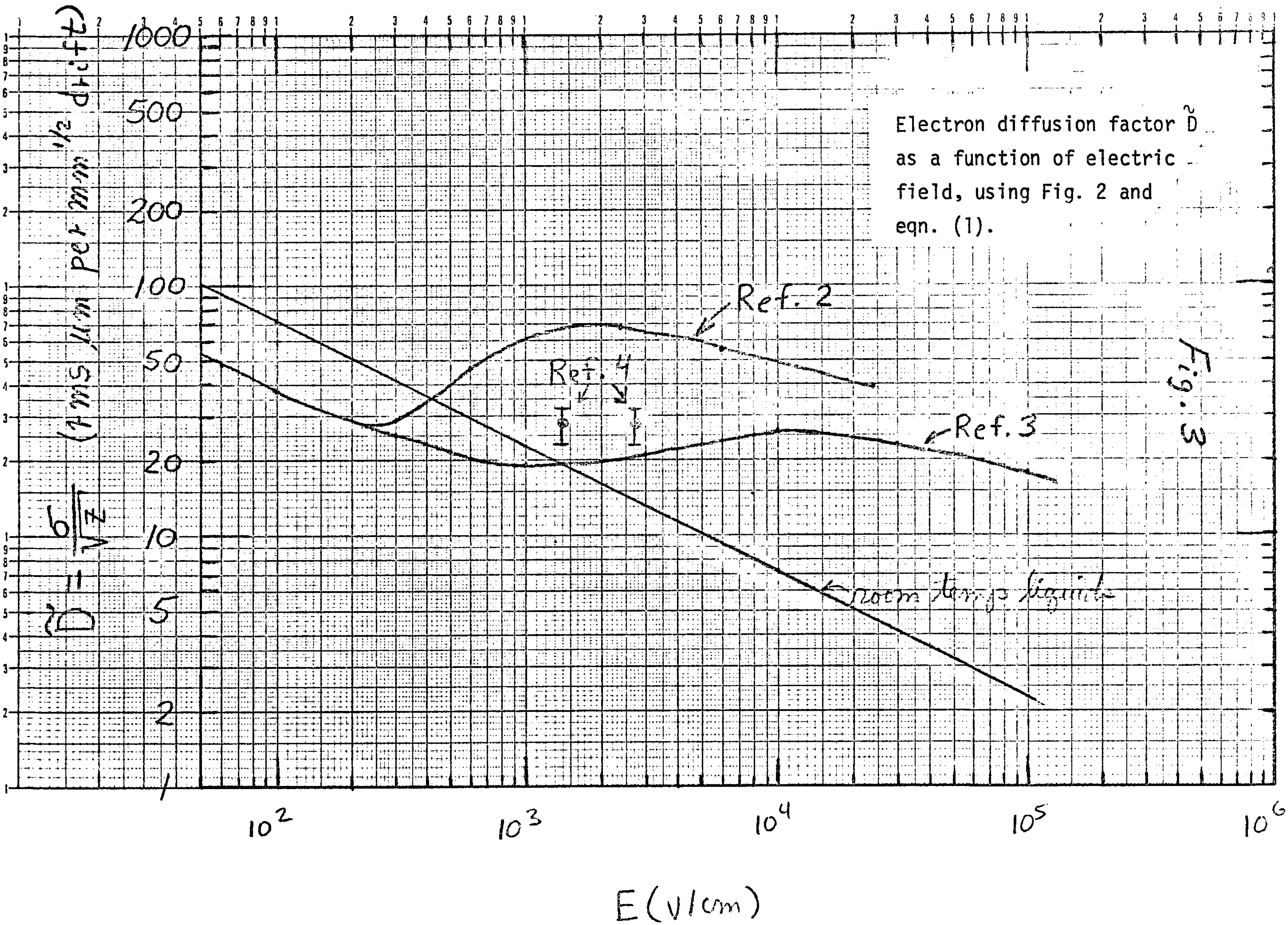


Fig. 3

Electron diffusion factor $\tilde{D}$ as a function of electric field, using Fig. 2 and eqn. (1).

$$F(X,Y) = \int_0^{\Delta} \frac{\text{EXP}\left(-\frac{X^2+Y^2}{2[\sigma(Z)]^2}\right)}{\sqrt{2\pi}\,\Delta}\, dZ \qquad (5)$$

where $\sigma(Z) = \frac{2D}{\mu}\frac{Z}{E}$ and Δ is the thickness of the detection medium. The electrons are detected by charge amplifiers connected to the anode strips.

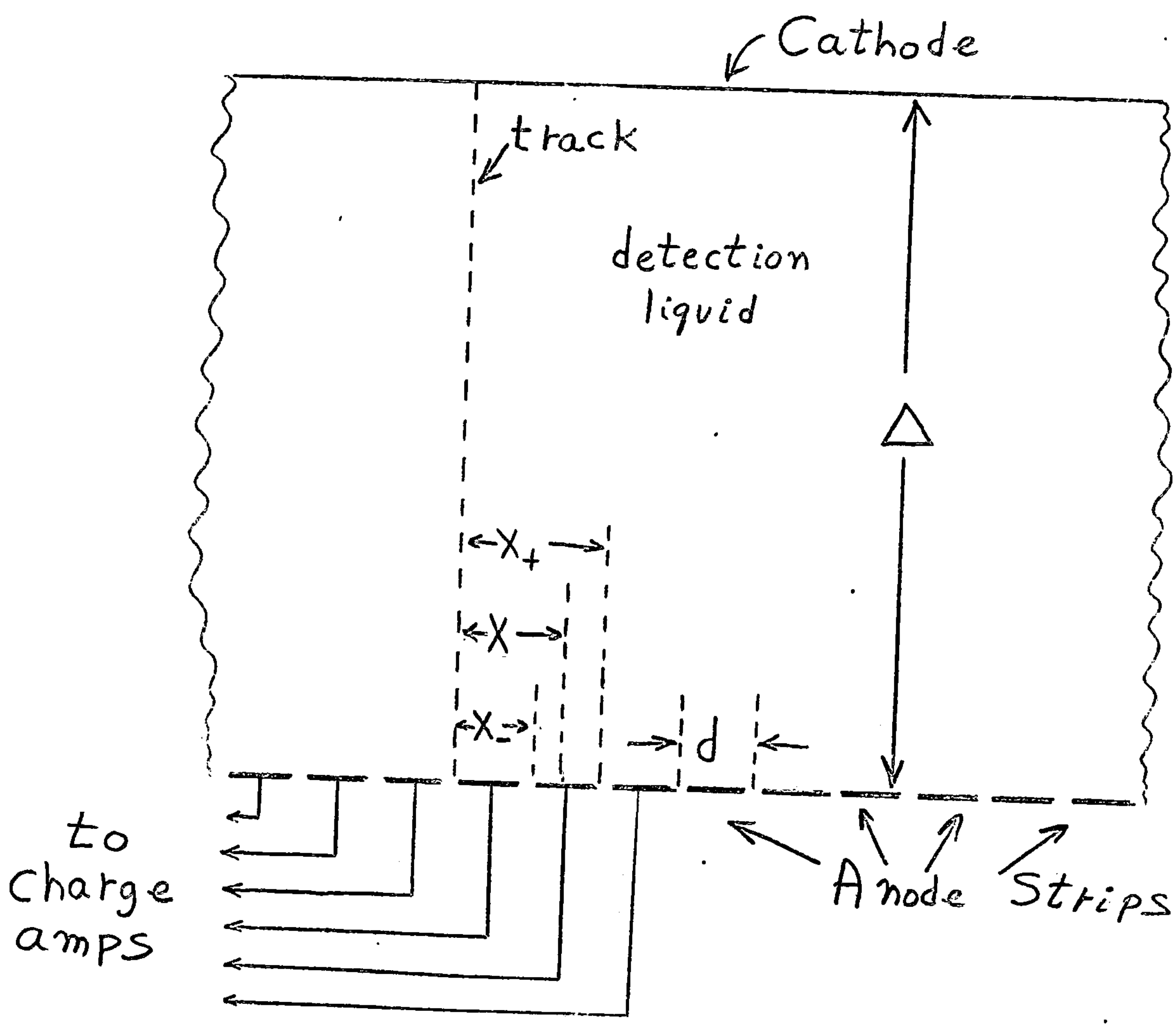


Fig. 4

The fractional charge collected on a strip is obtained by integrating eqn. (5) from $X_- = X - d/2$ to $X_+ = X + d/2$. where d is the strip spacing and X is the distance from the track to the center of the strip.

$$P(X) = \int_{X_-}^{X_+} dX' \int_{-\infty}^{\infty} F(X', Y')\, dY' \tag{6}$$

Note that

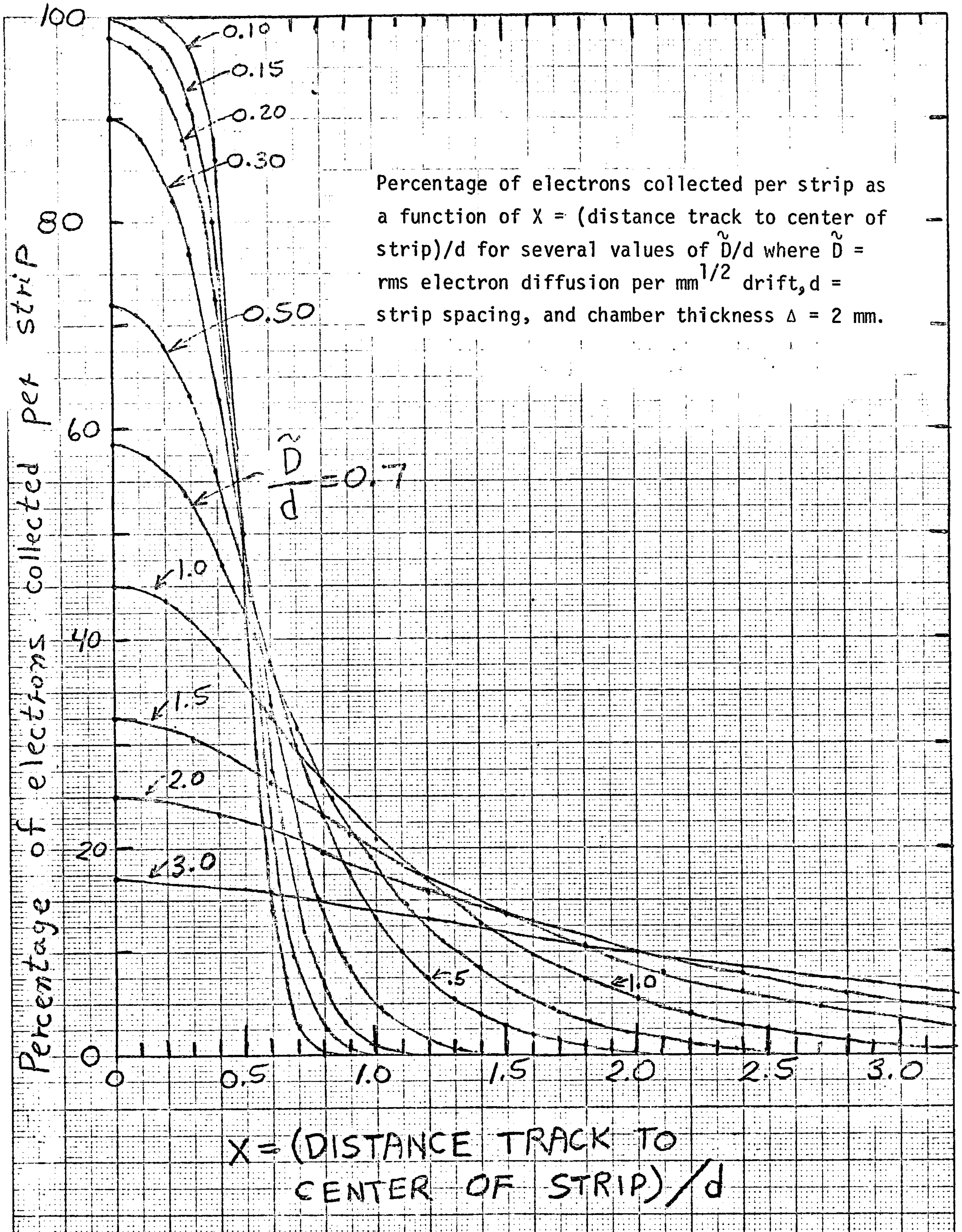
$$\int_{-\infty}^{\infty} dX' \int_{-\infty}^{\infty} F(X', Y')\, dY' = 1. \tag{7}$$

Curves of P vs. X are plotted Fig. 5 for various values of $\tilde{D}/d$, assuming $\Delta = 2$ mm.

Note that for tracks not quite normally incident the electrons may have an initial spread ΔY at right angles to the direction of the strips. To first order, this can be taken under consideration by using $d + \Delta Y$ as the effective strip width. For a chamber thickness $\Delta = 2$ mm, diffusion factor $\tilde{D} = 30\ \mu m/mm^{1/2}$, and a strip spacing $d = 20 \mu m$, this correction is small for angles below 5 mr.

3.2 Pulse Height Distributions

In Fig. 6 we plot pulse height distributions as a function of $\tilde{D}/d$, assuming that <u>all</u> values of track position X are equally populated. An amplifier noise of rms = 5% of the full pulse height has been folded in. Each curve has a vertical bar corresponding to the value X = 0.5 (i.e., when the track passes directly between two strips) in order to give a feeling for the distributions that result when only the interval X = (0, 0.5) is populated.



Percentage of electrons collected per strip as a function of X = (distance track to center of strip)/d for several values of $\tilde{D}/d$ where $\tilde{D}$ = rms electron diffusion per $mm^{1/2}$ drift, d = strip spacing, and chamber thickness Δ = 2 mm.

Fig 5

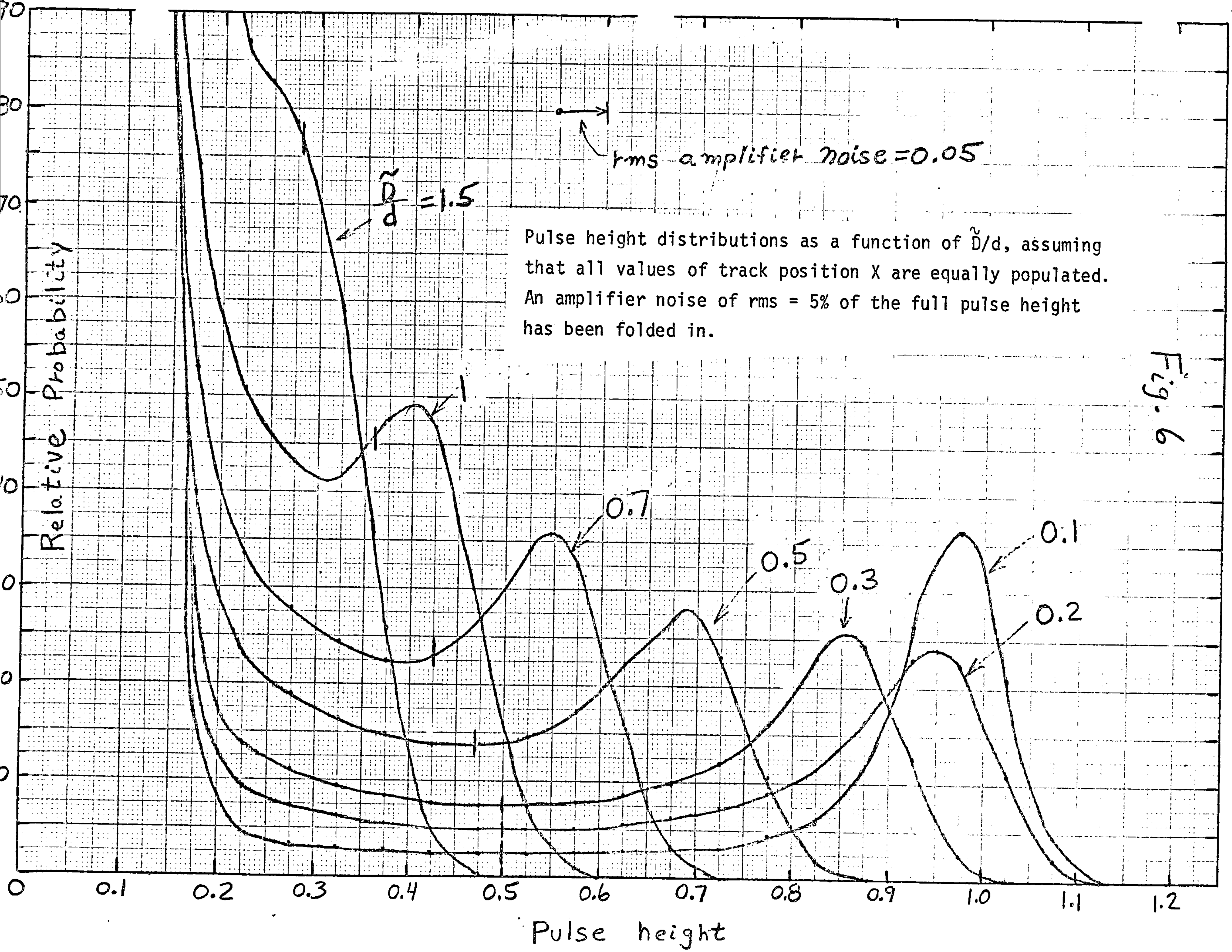


Fig. 6

Pulse height distributions as a function of $\tilde{D}/d$, assuming that all values of track position X are equally populated. An amplifier noise of rms = 5% of the full pulse height has been folded in.

4. Reduction in Pulse Height due to the Ar^+ Ions Along the Track

Typically, $\sim 1\mu s$ after the passage of the track, all ionization electrons have reached the anode. Not all of this charge is observed by the charge amplifiers, however, because some of the electrons are electrostatically bound to the slow Ar^+ ions.[5] In approximately 1 sec these electrons are released as the Ar^+ ions drift toward the cathode, but the pulse filtering commonly used in charge amplifiers does not permit such a slow pulse component to be observed.

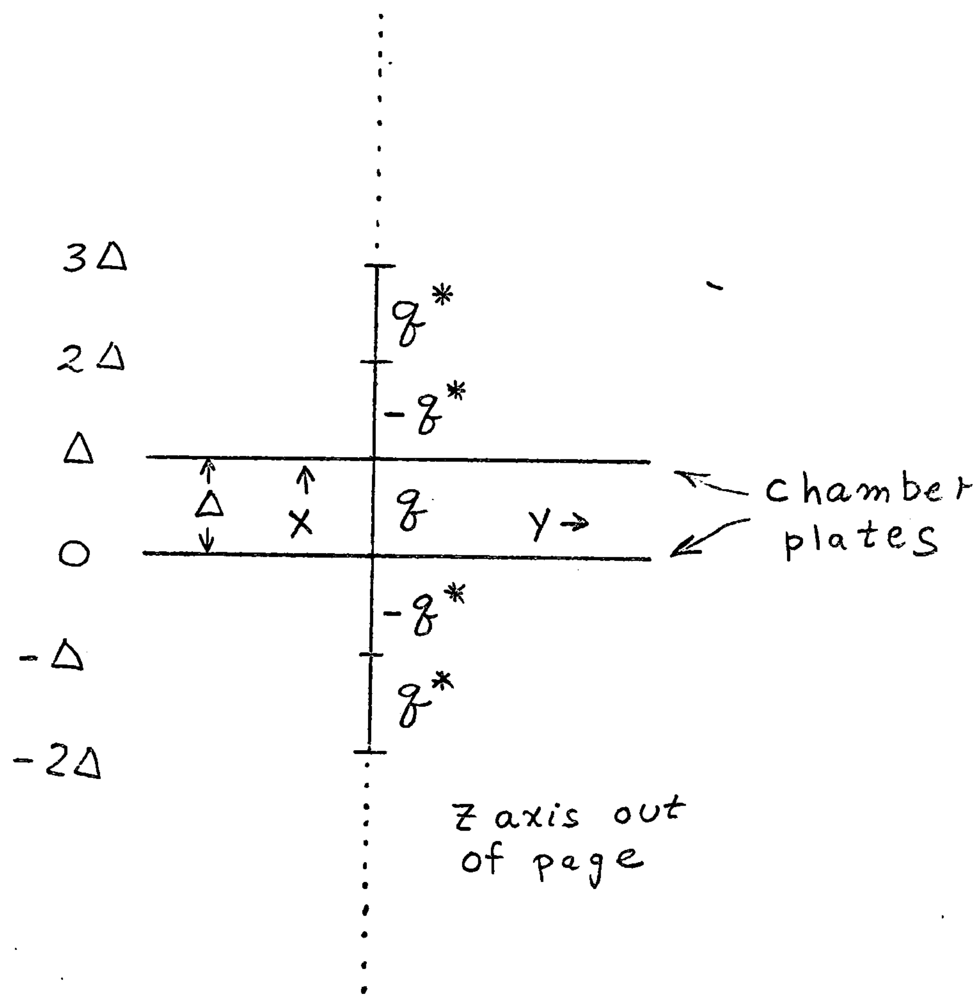
We calculate this effect by assuming a line of charge perpendicular to and extending between two parallel conducting plates (spacing Δ). This charge produces an electric field that has three properties of interest: (1) The field enters the plates normal to their surface; (2) Integrating the field over the surfaces of both plates yields a value that is related to the charge via Gauss' theorem, and (3) Integrating the field over a portion of the surface of one of the plates (such as over one anode strip) yields a value proportional to the bound charge whose value we seek.

To calculate the electric field at the surface of one plate, we use the method of image charges as shown in Fig. 7.

SUBJECT

NAME

DATE



q - space charge in chamber

±q* - image charges

Fig. 7

$$E(Y,Z) = 2 \sum_{i=0}^{\infty} Q\,(-1)^i \int_{X_i}^{X_{i+1}} \frac{X\,dX}{(X^2 + Y^2 + Z^2)^{3/2}} \tag{8}$$

$$= 2 \sum_{i=0}^{\infty} Q\,(-1)^i \left[(X_i^{\,2} + Y^2 + Z^2)^{-1/2} - (X_{i+1}^{\,2} + Y^2 + Z^2)^{-1/2}\right]$$

where $X_i = i \cdot \Delta$ and the track is assumed to pass through $Y = 0$ $Z = 0$. The fraction of charge f bound to a strip of width d centered at $Y = 0$ is found by integrating $E\,(Y, Z)$ over the strip and dividing by the integral of $E\,(Y,Z)$ over both plates:

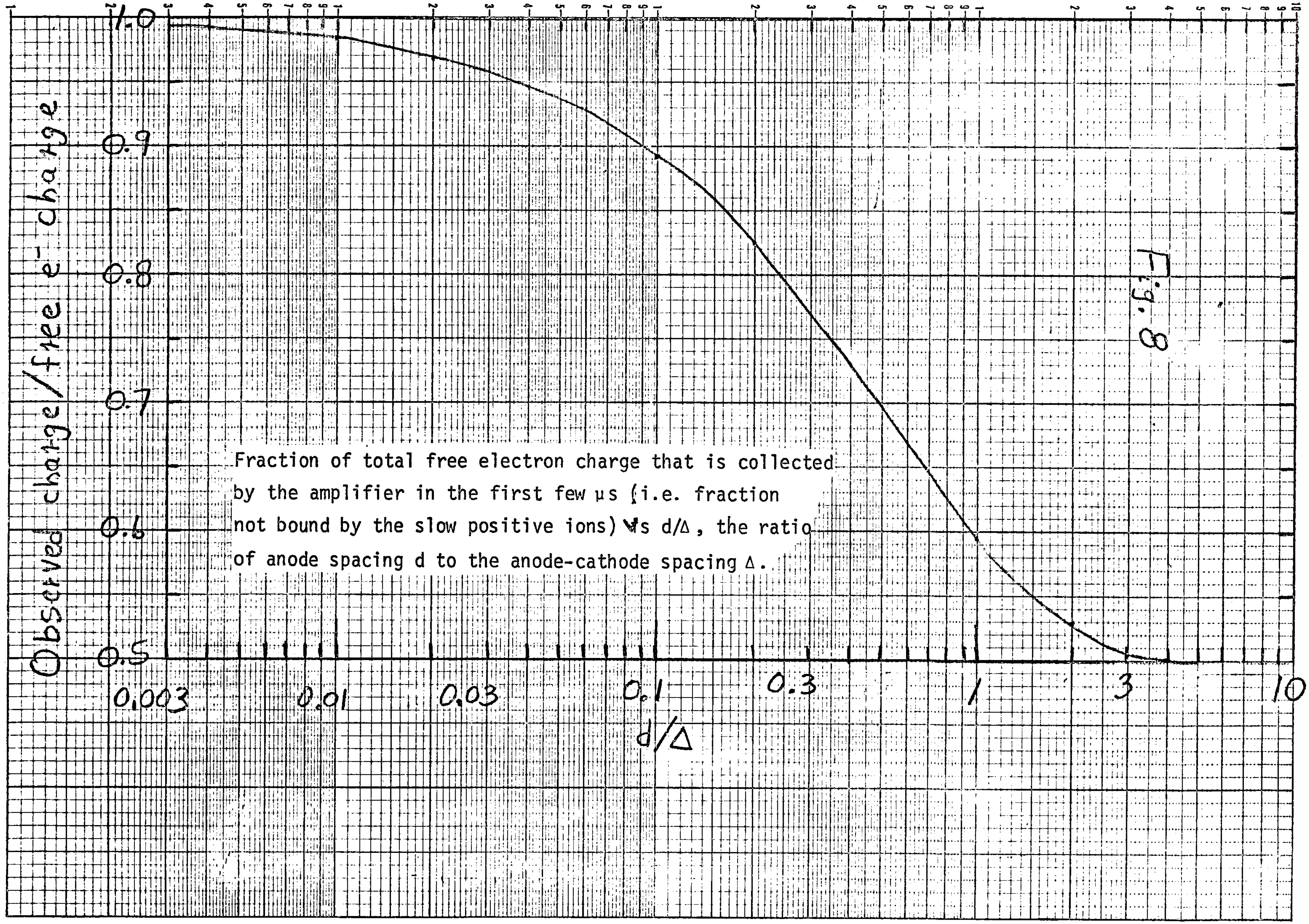
$$f = \frac{\int_0^{\infty} dZ \int_0^{d/2} E\,(Y, Z)\,dY}{2\int_0^{\infty} dZ \int_0^{\infty} E\,(Y, Z)\,dY}$$

$$= \frac{\int_0^{d/2} g\,(Y)\,dy}{2\int_0^{\infty} g\,(Y)\,dy}$$

The function $g\,(Y)$ is given by a very slowly convergent series:

$$g(Y) = \int_0^{\infty} E(Y,Z)\,dZ = \sum_{i=0}^{\infty} (-1)^i\, 2Q \ln\left[\frac{X_{i+1}^{\,2} + Y^2}{X_i^{\,2} + Y^2}\right]$$

In Fig. 8, we plot the observed charge $(1 - f)$ vs. d/Δ. Note that for small values of d/Δ nearly all the charge is observed and for large values of d/Δ only one half of the charge is observed.



Fraction of total free electron charge that is collected by the amplifier in the first few μs (i.e. fraction not bound by the slow positive ions) vs d/Δ, the ratio of anode spacing d to the anode-cathode spacing Δ.

SUBJECT

NAME

DATE

I am indebted to ~~Hiam~~ Haim Zaklad and Bernard Sadoulet for helpful discussions.

REFERENCES

1. W. F. Schmidt and A. O. Allen, J. Chem. Phys. 52, 4788 (1970); G. Bakale and W. F. Schmidt, Zeitschrift fur Naturforshung, 28a, 511 (1973); G. Gakale and W. F. Schmidt, Chem. Phys. Lett. 22. 164 (1973).
2. R. W. Warren and J. H. Parker, Phys. Rev. 128, 2661 (1962).
3. J. Lekner, Phys. Rev. 158, 130 (1967).
4. S. E. Derenzo, A. R. Kirschbaum, P. H. Eberhard, R. R. Ross, and F. T. Solmitz, Lawrence Berkeley Laboratory Report No. LBL-3058 (July 1974), to be published in Nuclear Instruments and Methods.
5. We thank Bernard Sadoulet for pointing out this effect to us.