

Longitudinal diffusion of electrons in electrostatic fields in gases

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MS. received 30th October 1968, in revised form 24th February 1969

Abstract. Expressions have been obtained for the longitudinal and lateral diffusion coefficients for electrons moving in electrostatic fields in gases, from the solution of the Boltzmann equation. Numerical results are quoted for the case of a collision frequency varying as the velocity raised to a power γ , on the assumption of no thermal motions of the gas molecules, and for the case of a constant collision cross section, with thermal motions included. The diffusion is found usually to be strongly anisotropic, in agreement with experimental measurements of Hurst and Parks in 1966 and Wagner, Davis and Hurst in 1967.

1. Introduction

The free diffusion of electrons in electrostatic fields in gases, with no magnetic field present, has usually been assumed to be isotropic. Theories developed for the electron diffusion under this assumption, by Huxley (1960) and others, do satisfactorily explain experimental measurements of the *lateral* diffusion coefficient (Huxley and Crompton 1962, Frost and Phelps 1962, and others). Measurements by Hurst and Parks (1966) and Wagner *et al.* (1967) on the *longitudinal* diffusion of electrons show, however, that the electron diffusion tensor generally is strongly anisotropic, and usually with the longitudinal diffusion coefficient smaller than the lateral one.

In the present work expressions are obtained for the components of the diffusion tensor from the Boltzmann equation, and physical arguments are advanced which explain the anisotropy.

The results of this work are in good agreement with recent theoretical work of Parker and Lowke (1968, 1969).

2. Theory

2.1. General considerations

The diffusion tensor \mathbf{D} for particles moving with a steady drift velocity $\langle \mathbf{v} \rangle$ can be expressed as the correlation function between the random velocity $\mathbf{v}^*(t)$ and the random displacement $\mathbf{r}^*(t, T)$ in the limit $T \rightarrow \infty$ (Skullerud 1969):

$$\mathbf{v}^*(t) = \mathbf{v}(t) - \langle \mathbf{v} \rangle \tag{1}$$

$$\mathbf{r}^*(t, T) = \int_t^{t+T} \mathbf{v}^*(t') dt' \tag{2}$$

$$\mathbf{D} = \left\langle \mathbf{v}^*(t) \int_t^\infty \mathbf{v}^*(t') dt' \right\rangle. \tag{3}$$

The drift velocity does not enter into the expression for the *lateral* diffusion coefficient D_\perp :

$$D_\perp = \left\langle v_x(t) \int_t^\infty v_x(t') dt' \right\rangle \tag{4}$$

v_x being one of the velocity components perpendicular to the drift direction.

In the case of electron motion the small ratio m/M between the mass of an electron and the mass of a gas molecule justifies the use of two simplifying assumptions:

(i) The magnitude of the velocity is maintained nearly constant throughout many collisions.

(ii) The persistence of velocity $m/(M+m)$ is small, and may be put equal to zero when considering the motion perpendicular to the drift motion.

The first assumption permits the use of a constant mean free time $\tau_M(v)$ for momentum transfer when calculating D_\perp from (4). The second assumption then leads to the substitution $v_x(t)\tau_M(v)$ for the integral in equation (4). The expression for D_\perp then assumes the form

$$D_\perp = \langle v_x^2 \tau_M(v) \rangle. \quad (5)$$

Inserting $\tau_M(v) = 1/\nu_M(v)$, ν_M being the momentum-transfer collision frequency, and averaging over a nearly isotropic velocity distribution function yields the well-known expression

$$D_\perp = \frac{1}{3} \left\langle \frac{v^2}{\nu_M(v)} \right\rangle. \quad (6)$$

For the longitudinal diffusion coefficient D_\parallel , equation (1) assumes the form

$$D_\parallel = \left\langle \{v_z(t) - \langle v_z \rangle\} \int_t^\infty \{v_z(t') - \langle v_z \rangle\} dt' \right\rangle \quad (7)$$

v_z being the velocity component in the drift direction. $\langle v_z \rangle$ will usually be small compared with v_z , except for the lowest-energy electrons, but can, nevertheless, *not* be neglected. This is due to the small value of the ratio $\xi = \tau_M/\tau_e = \nu_e/\nu_M$ between the energy- and momentum-transfer collision frequencies, which allows an electron to move with a momentum-transfer frequency $\nu_M(v)$ different from the mean momentum-transfer frequency $\langle \nu_M(v) \rangle$ throughout many momentum-transfer collisions.

An instantaneous drift velocity $v_d(v)$ different from $\langle v_z \rangle$ can therefore be maintained by a given energy group for a long time, and the resulting spread in drift velocities for the different energy groups should be expected to give a contribution to D_\parallel (positive or negative) of the order of magnitude

$$\begin{aligned} (D_\parallel)_d &\sim \frac{1}{3} \langle \{v_d(v) - \langle v_z \rangle\}^2 \tau_e \rangle \\ &= \frac{1}{3} \left\langle \{v_d(v) - \langle v_z \rangle\}^2 \frac{1}{\xi \nu_M} \right\rangle \\ &\sim \frac{1}{3} \frac{\langle v_z \rangle^2}{\langle \xi \nu_M \rangle}. \end{aligned} \quad (8)$$

This can easily be seen to be of the same order of magnitude as the lateral diffusion coefficient D_\perp , except for the special case of a velocity-independent collision frequency, where $v_d(v) = \langle v_z \rangle$ for all v .

2.2. The Boltzmann equation

The diffusion problem was formulated in terms of the Boltzmann equation by Wannier (1953), who calculated the longitudinal diffusion coefficient for the case of a mass ratio $m/M = 1$, solid-sphere interaction and strong electric fields. Wannier's method, adapted to the problem of electron diffusion, is used in this work. In this section his formulation of the diffusion problem will be briefly resumed.

The Boltzmann equation for the electron (or ion) velocity distribution function $f(\mathbf{r}, \mathbf{v}, t)$ may be written

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{d\mathbf{f}}{d\mathbf{r}} + \mathbf{a} \cdot \frac{\partial f}{\partial \mathbf{v}} = C(f) \quad (9)$$

$C(f)$ being the collision integral and \mathbf{a} the acceleration of an electron in an electrostatic field \mathbf{E} :

$$\mathbf{a} = -\frac{e\mathbf{E}}{m}. \quad (10)$$

Wannier solves equation (9) in two stages. In the first, only the term $\mathbf{a} \cdot \partial f / \partial \mathbf{v}$ is retained on the left. In the second, the full equation is used, with the new terms treated as perturbations.

The first stage describes an electron gas with a uniform number density n and no time dependence. The solution of equation (9) may then be written

$$f(\mathbf{v}) = nh(\mathbf{v}) \quad (11)$$

$h(\mathbf{v})$ being a velocity distribution function normalized to unity:

$$\int h(\mathbf{v}) d^3v = 1. \quad (12)$$

The drift velocity $\langle \mathbf{v} \rangle = \langle \mathbf{v}_z \rangle$ is normally defined as the average velocity in the absence of density gradients:

$$\langle \mathbf{v} \rangle = \int \mathbf{v}_z h(\mathbf{v}) d^3v. \quad (13)$$

In the second stage the influence of a density gradient is included, giving rise to a diffusion velocity \mathbf{v}_{diff} in addition to the drift velocity:

$$\int \mathbf{v} f(\mathbf{r}, \mathbf{v}, t) d^3v = n(\mathbf{r}, t)(\langle \mathbf{v} \rangle + \mathbf{v}_{\text{diff}}). \quad (14)$$

For sufficiently small relative density gradients \mathbf{k}

$$\mathbf{k} = n^{-1} \frac{\partial n}{\partial \mathbf{r}} \quad (15)$$

the diffusion velocity is proportional to \mathbf{k} , with the diffusion tensor as the proportionality factor:

$$\mathbf{v}_{\text{diff}} = -\mathbf{D} \cdot \mathbf{k}. \quad (16)$$

To calculate the diffusion tensor, Wannier assumes \mathbf{k} to be constant, i.e. he assumes a density

$$n(\mathbf{r}, t) = n_0 \exp\{\mathbf{k} \cdot (\mathbf{r} - \langle \mathbf{v} \rangle t)\} \quad (17)$$

and for $f(\mathbf{r}, \mathbf{v}, t)$ uses the substitution

$$f(\mathbf{r}, \mathbf{v}, t) = n(\mathbf{r}, t)\{h(\mathbf{v}) + d(\mathbf{v})\}. \quad (18)$$

$d(\mathbf{v})$ is the perturbation on the velocity distribution function due to the density gradient, and is normalized to zero:

$$\int d(\mathbf{v}) d^3v = 0. \quad (19)$$

The diffusion velocity is

$$\mathbf{v}_{\text{diff}} = \int \mathbf{v} d(\mathbf{v}) d^3v. \quad (20)$$

Insertion of equation (18) in the Boltzmann equation (9) yields the following equation for $d(\mathbf{v})$:

$$\mathbf{a} \cdot \frac{\partial d}{\partial \mathbf{v}} - C(d) = -\mathbf{k} \cdot (\mathbf{v} - \langle \mathbf{v} \rangle)\{h(\mathbf{v}) + d(\mathbf{v})\}. \quad (21)$$

If $h(\mathbf{v}) \gg d(\mathbf{v})$, $d(\mathbf{v})$ and hence \mathbf{v}_{diff} are seen to be proportional to \mathbf{k} , in agreement with equation (16), as $C(d)$ is linear in d .

If we consider diffusion only in the direction of the electric field, \mathbf{k} may be chosen in the z direction. The $d(\mathbf{v})$ term is omitted on the right of equation (21), and a reduced velocity distribution function

$$g(\mathbf{v}) = \frac{d(\mathbf{v})}{k} \tag{22}$$

is substituted instead of $d(\mathbf{v})$. The equation for $g(\mathbf{v})$ becomes

$$a \frac{\partial g}{\partial v_z} - C(g) = -(v_z - \langle v_z \rangle) h(\mathbf{v}). \tag{23}$$

$h(\mathbf{v})$ is determined by the homogeneous form of equation (23). The longitudinal diffusion coefficient is

$$D_{\parallel} = - \int v_z g(\mathbf{v}) d^3v. \tag{24}$$

The lateral diffusion coefficient is given by equations similar to equations (23) and (24):

$$a \frac{\partial g'}{\partial v_z} - C(g') = -v_z h(\mathbf{v}) \tag{25}$$

$$D_{\perp} = - \int v_z g'(\mathbf{v}) d^3v. \tag{26}$$

It is, however, easier to calculate D_{\perp} by means of equation (5), and equations (25) and (26) will accordingly not be used here.

2.3. Solution of the Boltzmann equation

The functions $g(\mathbf{v})$ and $h(\mathbf{v})$ are expanded in Legendre polynomials:

$$(4\pi)^{-1} g(\mathbf{v}) = g_0(v) + g_1(v) P_1(\cos \theta) + \dots \tag{27}$$

$$(4\pi)^{-1} h(\mathbf{v}) = h_0(v) + h_1(v) P_1(\cos \theta) + \dots \tag{28}$$

θ being the angle between \mathbf{v} and \mathbf{a} .

The expansion (27) is inserted in equation (23), which is multiplied with

$$P_l(\cos \theta) d(\cos \theta)$$

and subsequently integrated to yield an infinite set of coupled equations:

$$\begin{aligned} C_l(g) - \frac{al}{2l+1} \left(\frac{dg_{l-1}}{dv} - (l-1) \frac{g_{l-1}}{v} \right) - a \frac{l+1}{2l+3} \left(\frac{dg_{l+1}}{dv} + (l+2) \frac{g_{l+1}}{v} \right) \\ = \frac{l}{2l+1} v h_{l-1} - \langle v_z \rangle h_l + \frac{l+1}{2l+3} v h_{l+1}. \end{aligned} \tag{29}$$

The functions $h_l(v)$ are determined by the homogeneous form of equation (29).

In the case of electron motion the velocity distribution function will usually be nearly isotropic, and equations (29) can be simplified by the assumptions

$$\frac{g_l}{g_{l-1}} \sim \frac{h_l}{h_{l-1}} \sim \frac{\langle v_z \rangle}{v} \sim \left(\frac{m}{M} \right)^{1/2} \ll 1 \tag{30}$$

(see, for example, Wannier 1953). The assumptions (30) lead to the following approximate equations for g_0 and g_1 :

$$C_0(g) - \frac{a}{3} \left(\frac{dg_1}{dv} + 2 \frac{g_1}{v} \right) = - \langle v_z \rangle h_0 + \frac{1}{3} v h_1 \tag{31}$$

$$C_1(g) - a \frac{dg_0}{dv} = v h_0 \tag{32}$$

and similar homogeneous forms for h_0 and h_1 . If we specialize to the case of *elastic* collisions only, the collision integrals $C_0(g)$ and $C_1(g)$ attain the Davydov form

$$C_0(g) = \frac{1}{2v^2} \frac{d}{dv} \left\{ v^3 \nu_M \xi \left(g_0 + \frac{kT}{mv} \frac{dg_0}{dv} \right) \right\} \quad (33)$$

$$C_1(g) = -\nu_M g_1 \quad (34)$$

(Shkarofsky *et al.* 1966, equation (3-58)), ν_M being the momentum-transfer collision frequency, k the Boltzmann constant, T the gas temperature and ξ the ratio between the energy- and momentum-transfer frequencies:

$$\xi = \frac{\nu_e}{\nu_M} = \frac{2m}{M}. \quad (35)$$

Equations (33) and (34) are valid also when inelastic collisions are of importance, as long as the inelastic energy losses are small compared with kT (Shkarofsky *et al.* 1966). This will be the case, for example, for rotational energy losses in nitrogen at room temperature. A velocity-dependent $\xi(v)$ must then be used. Equations (31) and (32) may now be written

$$\frac{a}{3v^2} \frac{d}{dv} (g_1 v^2) - \frac{1}{2v^2} \frac{d}{dv} \left\{ v^3 \nu_M \xi \left(g_0 + \frac{kT}{mv} \frac{dg_0}{dv} \right) \right\} = v_a h_0 - \frac{1}{3} v h_1 \quad (36)$$

$$a \frac{dg_0}{dv} + \nu_M g_1 = -v h_0. \quad (37)$$

After multiplication with v^2 , equation (36) may be integrated directly. If we insert g_1 from equation (37), the following equation for g_0 is then obtained:

$$\begin{aligned} \left(\frac{1}{3} \left(\frac{a}{\nu_M} \right)^2 + \frac{\xi kT}{2m} \right) \frac{dg_0}{dv} + \frac{1}{2} v \xi g_0 &= \frac{1}{3v} \left(\frac{a}{\nu_M} \right) v h_0 \\ &+ \frac{1}{3v^2 \nu_M} \int_0^v w^3 h_1(w) dw - \frac{\langle v_z \rangle}{v^2 \nu_M} \int_0^v w^2 h_0(w) dw. \end{aligned} \quad (38)$$

$h_0(v)$ is given by the homogeneous form of equation (38), which has as solution the Davydov distribution (Davydov 1935)

$$h_0(v) = A \exp \left[-\frac{3}{2} \int_0^v v \xi dv \left\{ \left(\frac{a}{\nu_M} \right)^2 + \frac{3}{2} \xi \frac{kT}{m} \right\}^{-1} \right]. \quad (39)$$

A is a normalization factor, which is determined by the condition

$$\int_0^\infty h_0(v) v^2 dv = 1$$

h_1 is obtained from the homogeneous form of equation (37):

$$h_1(v) = -\frac{a}{\nu_M} \frac{dh_0}{dv}. \quad (40)$$

For $g_0(v)$ we now make the substitution

$$g_0(v) = y(v) h_0(v) \quad (41)$$

which is inserted into equation (38). Further, the integral over h_1 is transformed to an integral over h_0 by the use of equation (40) and a partial integration:

$$\frac{1}{3} \int_0^v h_1(w) w^3 dw = -\frac{a}{3v} v^3 h_0(v) + \int_0^v v_a(w) h_0(w) w^2 dw. \quad (42)$$

Here the substitution

$$v_d(v) = \frac{1}{3v^2} \frac{d}{dv} \left(\frac{a}{\nu_M} v^3 \right) \quad (43)$$

has been made, $v_d(v)$ being the instantaneous drift velocity for electrons with a velocity of magnitude v (Huxley 1960).

The equation for $y(v)$ now becomes

$$a \frac{dy}{dv} = \left\{ \left(\frac{a}{\nu_M} \right)^2 + \frac{3}{2} \xi \frac{kT}{m} \right\}^{-1} \left[-2v \left(\frac{a}{\nu_M} \right)^2 + 3 \frac{a}{\nu_M} \frac{1}{h_0 v^2} \int_0^v \{v_d(w) - \langle v_z \rangle\} h_0(w) w^2 dw \right]. \quad (44)$$

It will not be necessary to solve this equation for y to find the diffusion coefficient $D_{||}$, given by equation (24). The use of (27) and (37) yields

$$\begin{aligned} D_{||} &= - \int v_z g(v) d^3v = - \frac{1}{3} \int_0^\infty v^3 g_1(v) dv \\ &= \frac{1}{3} \int_0^\infty \frac{v^2}{\nu_M} h_0(v) v^2 dv + \frac{1}{3} \int_0^\infty \frac{a}{\nu_M} v^3 dg_0. \end{aligned} \quad (45)$$

The first term on the right is recognized as the lateral diffusion coefficient D_{\perp} (see equation (6)):

$$D_{\perp} = \frac{1}{3} \int_0^\infty \frac{v^2}{\nu_M} h_0(v) v^2 dv. \quad (46)$$

The longitudinal diffusion coefficient may thus be written

$$\begin{aligned} D_{||} &= D_{\perp} + D_1 \\ D_1 &= \frac{1}{3} \int_0^\infty \frac{a}{\nu_M} v^3 dg_0. \end{aligned} \quad (47)$$

A partial integration transforms the expression for D_1 to

$$D_1 = - \frac{1}{3} \int_0^\infty g_0 \frac{d}{dv} \left(\frac{a}{\nu_M} v^3 \right) dv = - \int_0^\infty g_0 v_d(v) v^2 dv. \quad (48)$$

Insertion of $g_0(v) = y(v)h_0(v)$ yields, after a partial integration,

$$\begin{aligned} D_1 &= - \int_0^\infty y(v) h_0(v) v_d(v) v^2 dv \\ &= -y(\infty) \langle v_z \rangle + \int_0^\infty \frac{dy}{dv} dv \int_0^v h_0(w) v_d(w) w^2 dw. \end{aligned} \quad (49)$$

The normalization condition equation (19) is used to find $y(\infty)$. With the use of equations (22), (27) and (41), and after a partial integration, equation (19) yields

$$y(\infty) = \int_0^\infty \frac{dy}{dv} dv \int_0^v h_0(w) w^2 dw. \quad (50)$$

An expression for the lateral diffusion coefficient may now be written down, using equations (44), (47), (49) and (50):

$$\begin{aligned} D_{||} &= D_{\perp} + \frac{1}{a} \int_0^\infty \left\{ \left(\frac{a}{\nu_M} \right)^2 + \frac{3}{2} \xi \frac{kT}{m} \right\}^{-1} \\ &\quad \times \left[-2v \left(\frac{a}{\nu_M} \right)^2 + 3 \left(\frac{a}{\nu_M} \right) \{h_0(v) v^2\}^{-1} I(v) \right] I(v) dv \end{aligned} \quad (51)$$

$$I(v) = \int_0^v \{v_d(w) - \langle v_z \rangle\} h_0(v) v^2 dv. \quad (52)$$

D_{\perp} is given by equation (46), $h_0(v)$ by equation (39), $v_d(v)$ by equation (43) and $\langle v_z \rangle$ is

$$\langle v_z \rangle = \int_0^{\infty} v_d(v) h_0(v) v^2 dv. \quad (53)$$

3. Numerical calculations

The longitudinal and lateral diffusion coefficients have been calculated numerically from equations (46) and (51) for the following cases: (i) elastic collisions, a velocity-independent mean free path λ_M for momentum transfer ($\lambda_M = v/\nu_M$) and ratios a/T between the acceleration and the temperature varying from zero to infinity; (ii) elastic collisions, a collision frequency $\nu(v) \propto v^{\gamma}$ ($-1 < \gamma < 3$) and a temperature $T = 0$.

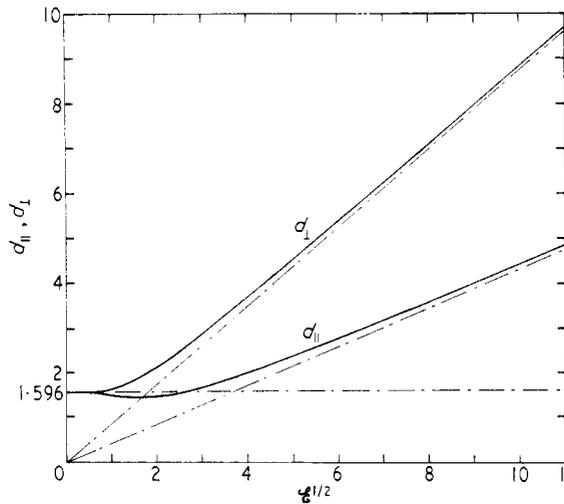


Figure 1. The dimensionless diffusion coefficients $d_{\parallel} = ND_{\parallel}\sigma_M(kT/m)^{-1/2}$ and $d_{\perp} = ND_{\perp}\sigma_M(kT/m)^{-1/2}$ as functions of the dimensionless field parameter $\mathcal{E} = (E/N) (e/kT\sigma_M) (M/m)^{1/2}$. Asymptotic values for large and small \mathcal{E} are shown as broken lines.

(i) The results for a velocity-independent mean free path are shown in figure 1, where the dimensionless quantities

$$d_{\parallel} = ND_{\parallel}\sigma_M \left(\frac{kT}{m} \right)^{-1/2} \quad (54)$$

$$d_{\perp} = ND_{\perp}\sigma_M \left(\frac{kT}{m} \right)^{-1/2}$$

are given as functions of the dimensionless field parameter

$$\mathcal{E} = \frac{E}{N} \frac{e}{kT\sigma_M} \left(\frac{M}{m} \right)^{1/2}. \quad (55)$$

N is the gas molecule number density and σ_M is the momentum-transfer cross section ($\sigma_M = 1/N\lambda_M$). For small values of \mathcal{E} , d_{\parallel} and d_{\perp} approach the same (thermal) limit

$$d_{\parallel}(\mathcal{E} = 0) = d_{\perp}(\mathcal{E} = 0) = 1.596. \quad (56)$$

The asymptotic values for large values of \mathcal{E} are

$$\begin{aligned} d_{\perp} &\rightarrow 0.8770 \mathcal{E}^{1.2} \\ &\mathcal{E} \rightarrow \infty \\ d_{\parallel} &\rightarrow 0.4306 \mathcal{E}^{1/2}. \end{aligned} \tag{57}$$

It may be noted as a curiosity that, at small values of \mathcal{E} , d_{\parallel} at first *decreases* with increasing \mathcal{E} , reaching a minimum value of 1.48 at $\mathcal{E} \simeq 2.5$.

(ii) The results for a collision frequency varying as the velocity raised to a power γ :

$$\nu_M(v) = \nu_M(v_0) \left(\frac{v}{v_0} \right)^{\gamma} \tag{58}$$

are shown in figure 2, where the dimensionless diffusion coefficient

$$\delta_{\perp} = D_{\perp} \left[\frac{v_0^2}{\nu_M(v_0)} \left\{ \left(\frac{M}{m} \right)^{1/2} \frac{a}{v_0 \nu_M(v_0)} \right\}^{(2-\gamma)/(1+\gamma)} \right]^{-1} \tag{59}$$

and the ratios D_{\parallel}/D_{\perp} are given as functions of γ . At $\gamma = 0$, $\delta_{\perp} = 1/3$ and $D_{\parallel}/D_{\perp} = 1$.

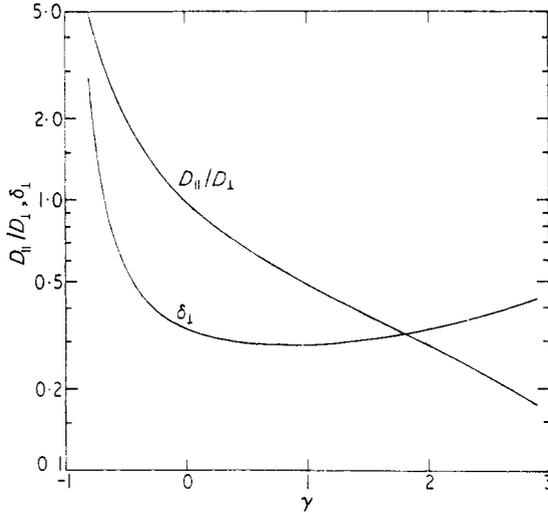


Figure 2. The dimensionless diffusion coefficient δ_{\perp} (equation (59)) and the ratio D_{\parallel}/D_{\perp} of the longitudinal to the lateral diffusion coefficient for the case of a strong electric field and a collision frequency $\nu_M \propto v^{\gamma}$, as functions of γ .

In the whole range investigated both δ_{\perp} and D_{\parallel}/D_{\perp} increase with decreasing γ , and approach infinity as $\gamma \rightarrow -1$. The reason for this divergence is evident when considering the Davydov distribution (39), which does *not* approach zero at large velocities for $\gamma \leq -1$. Physically this must be interpreted as due to the occurrence of runaway electrons, which makes the concept of a stationary velocity distribution function meaningless.

At $\gamma \geq 3$ the expressions which are used to calculate D_{\parallel} and D_{\perp} do also diverge. The reason for this is, however, not physical, but is due to the use of the simplifying assumptions (30). The divergence occurs in the low-energy limit $v \rightarrow 0$, where the assumptions

$$\frac{g_l}{g_{l-1}} \sim \frac{h_l}{h_{l-1}} \sim \frac{\langle v_z \rangle}{v} \ll 1$$

are no longer applicable.

4. Discussion

The expressions found in § 2.3 for the longitudinal and lateral diffusion coefficients are valid for elastic and weakly inelastic collisions only. When inelastic collisions with energy transfer larger than thermal energies but still small compared with the electron energies become important, these can easily be taken into account by a modification of the collision integral (33), as has been shown by Shkarofsky *et al.* (1966). If, however, the energy transfer in an inelastic collision is *not* small compared with the electron energies, as will be the case when vibrational excitation is of importance, then the Boltzmann equation cannot be transformed to a differential equation, but will show up as a difference differential equation. The solution of equations (31) and (32) will then be considerably more difficult, but can be performed by methods as used by Frost and Phelps (1962). Qualitatively the ratio D_{\parallel}/D_{\perp} must be expected to behave in the same way when inelastic collisions are of importance, as when only elastic collisions need to be taken into account.

The physical reason for the difference between the longitudinal and the lateral diffusion coefficients can be outlined as follows:

Let us consider the case of a collision frequency $\nu_M(v)$ *increasing* with the velocity. An electron diffusing against the \mathbf{a} direction will lose energy, and thus acquire a lower velocity, a lower collision frequency and an increased instantaneous drift velocity, which after some time (of the order $\tau_e = \tau_M(v)/\xi$) will have reduced the distance which the electron lagged behind the average electron position. Similarly an electron diffusing in the \mathbf{a} direction will gain energy, and acquire a lower instantaneous drift velocity. The longitudinal spread of a group of electrons will thus be diminished compared with the lateral spread due to a 'drift-phase-stabilization' mechanism.

We shall consider then the case of a collision frequency $\nu(v)$ *decreasing* with increasing velocity. In this case an electron which has diffused against the \mathbf{a} direction will have a lower instantaneous drift velocity, and an electron which has diffused in the \mathbf{a} direction has an increased instantaneous drift velocity. The longitudinal spread of a group of electrons will thus be enhanced compared with the lateral spread.

It should be noted that the concept of diffusion can only be used either when the time intervals considered are large compared with the autocorrelation time for the random velocity, or when the relative change of density is small over a distance l which an electron traverses during this time: $l \cdot \nabla n/n \ll 1$. For the longitudinal component of the random velocity this time is of the order of a mean free time for *energy* transfer, but for the lateral component it is of the order of a mean free time for *momentum* transfer, which is usually much less. In an experiment this means that the longitudinal spread of an electron swarm can usually be considered as being due to a diffusion process only when the electric potential difference between the electrodes is large compared with the average electron energy. At smaller potential differences the use of a diffusion equation to calculate the longitudinal motion of the swarm cannot be justified, except at energies near thermal energies. These restrictions do not apply to the lateral motion of the swarm.

5. Conclusion

Expressions have been obtained for the longitudinal and lateral diffusion coefficients for electrons moving in electrostatic fields in gases under the influence of elastic or weakly inelastic collisions. Numerical evaluations of these expressions, and also qualitative physical arguments, show that the lateral diffusion coefficient is larger than the longitudinal one when the momentum-transfer collision frequency ν_M increases monotonically with increasing velocity v , but is smaller than the longitudinal one when ν_M decreases with increasing v .

The results of this work explain the experimental measurements of Hurst and Parks (1966) and Wagner *et al.* (1967) on the longitudinal diffusion of electrons, and are in good agreement with recent theoretical works of Parker and Lowke (1968, 1969).

Acknowledgments

The author is grateful to Professor R. S. Sigmond and Mr. S. Stølen for several discussions and helpful suggestions.

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