Flux through two cosmic ray detectors

Consider two cosmic ray detectors, of dimensions $H \times W$ and $h \times w$, separated by a horizontal distance d and a vertical distance s. What is the flux of cosmic rays passing through both detectors?

Let us assume that the distance between the detectors is large enough that the range of angles subtended by a point on the first detector from the top to the bottom of the second detector is the same at all points on the detector.

The required range of vertical angles is given by:

$$tan(\theta_1) = \frac{d}{s - \frac{h}{2}}$$

$$tan(\theta_2) = \frac{d}{s + \frac{h}{2}}$$

$d =$ the horizontal distance between the detectors
$s =$ the vertical distance between the centres of the detectors
$h =$ the height of the second detector

The required range of horizontal angles is given by:

$$tan(\phi_1) = \frac{w}{2d}$$

$$tan(\phi_2) = -\frac{w}{2d}$$

$d =$ the horizontal distance between the detectors
$w =$ the width of the second detector

The flux passing through a vertical plane of area HW per unit time between the required angles is:

$$HWR \int_{tan^{-1}(\frac{w}{2d})}^{tan^{-1}(-\frac{w}{2d})} cos(\phi)d\phi \int_{tan^{-1}(\frac{d}{s+\frac{h}{2}})}^{tan^{-1}(\frac{d}{s-\frac{h}{2}})} sin(\theta)cos^2(\theta)sin(\theta)d\theta$$

$Cos(\phi)$ and $sin(\theta)$ are the projections of the direction of the ray on the plane, $cos^2(\theta)$ is the factor for the angular intensity of cosmic radiation, and R is the maximum intensity of the cosmic radiation.

Carrying out the integration:

$$HWR\int_{tan^{-1}(\frac{w}{2d})}^{tan^{-1}(\frac{w}{2d})} cos(\phi)d\phi \int_{tan^{-1}(\frac{d}{s+\frac{h}{2}})}^{tan^{-1}(\frac{d}{s-\frac{h}{2}})} sin(\theta)cos^2(\theta)sin(\theta)d\theta$$

$$= HWR[sin(\phi)]_{tan^{-1}(\frac{w}{2d})}^{tan^{-1}(\frac{w}{2d})} \int_{tan^{-1}(\frac{d}{s+\frac{h}{2}})}^{tan^{-1}(\frac{d}{s-\frac{h}{2}})} \frac{1}{4}sin^2(2\theta)d\theta$$

$$= \frac{HWRw}{\sqrt{d^2+\frac{w^2}{4}}}[\frac{\theta}{8} - \frac{1}{32}sin(4\theta)]_{tan^{-1}(\frac{d}{s+\frac{h}{2}})}^{tan^{-1}(\frac{d}{s-\frac{h}{2}})}$$

$$= \frac{HWRw}{\sqrt{d^2+\frac{w^2}{4}}}[\frac{\theta}{8} - \frac{1}{4}sin(\theta)cos^3(\theta) + \frac{1}{8}sin(\theta)cos(\theta)]_{tan^{-1}(\frac{d}{s+\frac{h}{2}})}^{tan^{-1}(\frac{d}{s-\frac{h}{2}})}$$

$$= \frac{HWRw}{\sqrt{d^2+\frac{w^2}{4}}}[\frac{1}{8}tan^{-1}(s_1) - \frac{1}{8}tan^{-1}(s_2) + \frac{s_1^3-s_1}{8(1+s_1^2)^2} - \frac{s_2^3-s_2}{8(1+s_2^2)^2}]$$

where $s_1 = \frac{d}{s-\frac{h}{2}}$ and $s_2 = \frac{d}{s+\frac{h}{2}}$.

Note that the last step uses the identities $cos(tan^{-1}(z)) = \frac{1}{\sqrt{1+z^2}}$ and $sin(tan^{-1}(z)) = \frac{z}{\sqrt{1+z^2}}$

Let us assume a vertical particle flux R = 88.7 particles per m^2 per second. This is the value for the flux of muons of energies above 0.3 GeV/c. The photon flux is negligible compared to the muon flux.(Grieder, P.K.F. *Cosmic Rays at Earth*. Amsterdam: Elsevier Science, 2001).

For two detectors both having dimensions 1.5m x 0.15m, separated horizontally by 12 feet (3.658m) and vertically by 6 feet (1.829m), this formula predicts a flux between the detectors of 0.040 particles per second.

However, we should be aware that in this case, because the size of the detectors is of the same order as the distances that separate them, our original approximation about the range of angles subtended may no longer be valid. Therefore let us carry out a more accurate calculation by integrating over the entire surface of the upper collector.

Consider a point (x,y), where x is the horizontal distance from the left side of the first detector and y is the vertical distance below the top of the first detector.

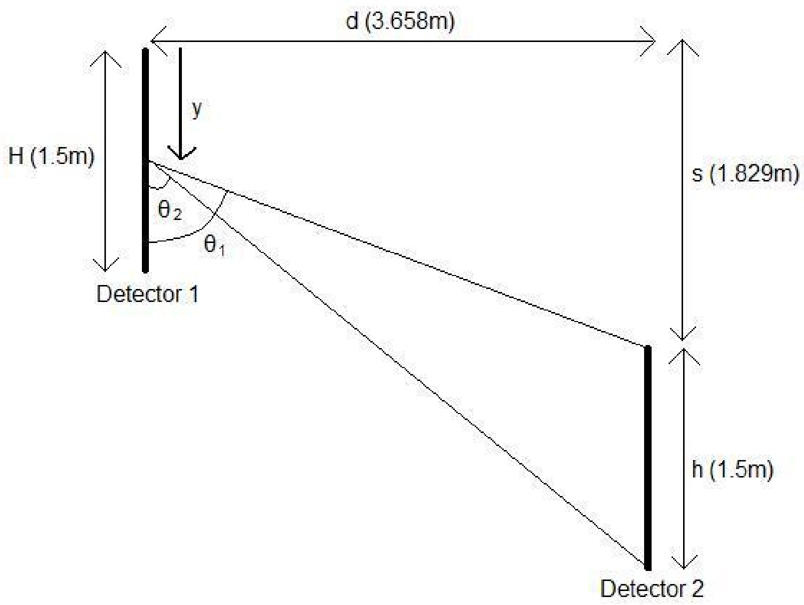

Fig. 1 : the range of vertical angles at y

The required range of vertical angles at (x,y) is given by:

$$tan(\theta_1) = \frac{d}{s - y}$$

$$tan(\theta_2) = \frac{d}{s + h - y}$$

d = the horizontal distance between the detectors
s = the vertical distance between the centres of the detectors
h = the height of the second detector

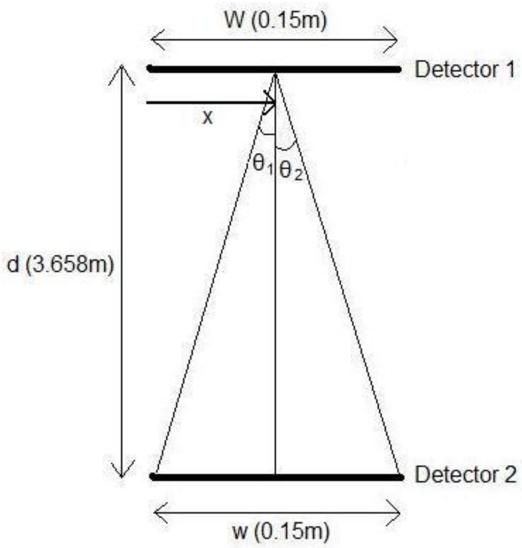

Fig. 2 : the range of horizontal angles at x

The required range of horizontal angles at x is given by:

$$tan(\phi_1) = \frac{x}{d}$$

$$tan(\phi_2) = \frac{w-x}{d}$$

$d =$ the horizontal distance between the detectors
$w =$ the width of the second detector

Then the flux between the appropriate angles passing through the area element dxdy at (x,y) is:

$$Rdxdy \int_{tan^{-1}(\frac{w-x}{d})}^{tan^{-1}(-\frac{x}{d})} cos(\phi)d\phi \int_{tan^{-1}(\frac{d}{s+h-y})}^{tan^{-1}(\frac{d}{s-y})} sin(\theta)cos^2(\theta)sin(\theta)d\theta$$

$$= Rdxdy(\frac{x}{\sqrt{d^2+x^2}} - \frac{w-x}{\sqrt{d^2+(w-x)^2}})$$

$$\times\frac{1}{8}(tan^{-1}(\frac{d}{s-y}) - tan^{-1}(\frac{d}{s+h-y}) + \frac{(\frac{d}{s-y})^3 - (\frac{d}{s-y})}{(1+\frac{d}{s-y}^2)^2} - \frac{(\frac{d}{s+h-y})^3 - (\frac{d}{s+h-y})}{(1+\frac{d}{s+h-y}^2)^2})$$

And the total flux over the detector between the appropriate angles is given by integrating over x and y:

$$R\int_0^w \left(\frac{x}{\sqrt{d^2+x^2}} - \frac{w-x}{\sqrt{d^2+(w-x)^2}}\right)dx$$

$$\times\frac{1}{8}\int_0^h \left(tan^{-1}\left(\frac{d}{s-y}\right) - tan^{-1}\left(\frac{d}{s+h-y}\right) + \frac{(\frac{d}{s-y})^3-(\frac{d}{s-y})}{(1+\frac{d}{s-y}^2)^2} - \frac{(\frac{d}{s+h-y})^3-(\frac{d}{s+h-y})}{(1+\frac{d}{s+h-y}^2)^2}\right)dy$$

$$= 2R(\sqrt{d^2+w^2}-d)$$

$$\times\frac{1}{8}\int_0^h \left(tan^{-1}\left(\frac{d}{s-y}\right) - tan^{-1}\left(\frac{d}{s+h-y}\right) + \frac{(\frac{d}{s-y})^3-(\frac{d}{s-y})}{1+(\frac{d}{s-y})^2)^2} - \frac{(\frac{d}{s+h-y})^3-(\frac{d}{s+h-y})}{(1+(\frac{d}{s+h-y})^2)^2}\right)dy$$

The y integral must be evaluated by numerical methods.

Again assume a vertical particle flux R = 88.7 particles per m^2 per second. For two detectors both having dimensions 1.5m x 0.15m, separated horizontally by 12 feet (3.658m) and vertically by 6 feet (1.829m), we can evaluate the y intergral by using C++ and dividing h into 1000 intervals: we obtain a result of 0.568337. Combining this with the formula obtained analytically, we predict a flux between the detectors of 0.0388 particles per second.