LArIAT: Phase 1 and 2 Theoretical Muon Incidence Rate

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The LArIAT project's different stages employ PMT-mounted scintillation counters for the purpose of measuring the flux of muons along different paths through the TPC. A critical initial step in ensuring that these counters are calibrated properly and function efficiently is a test of the agreement between their measured muon incidence rates under certain conditions and the theoretical values for such rates. These theoretical values are based on the dimensions of the detector and existing knowledge of ground-level cosmic muon flux.

Phase 1

LArIAT Phase 1 features two trapezoidal scintillation counter configurations situated at each end of the cryostat. Our goal is to calculate a theoretical muon incidence rate through both counters, assuming they are separated by the cryostat length d = 1.63 meters. The trapezoids' centers are separated both by a horizontal distance x = 0.778 m and a vertical distance s = 0.652 m, so that they appear near opposite "corners" of the cylindrical cryostat. We make the simplifying approximation that the trapezoids are rectangles of dimensions w x h = 0.298m x 0.504m.

Since flux is measured in units of $m^{-2}s^{-1}sr^{-1}$, a process determining the total number of particles per second through the two counters must account for the area of the counters and the solid angle effectively seen by the configuration. A method of doing so, found in "Flux through two cosmic ray detectors" by Emily Adlam (LArTPC docdb # 730), is shown in Figure 1, and begins by considering the cosmic muon flux and solid angle "viewed" by a differential area element on one detector. In the vertical direction, that differential area element dA receives particles from a continuous range of angles from θ_1 to θ_2. Although not shown, the horizontal direction is similar to Fig.1, with θ_1 and θ_2 replaced by the azimuthal angles ϕ_1 and ϕ_2. For a given θ and ϕ, the total number of particles crossing dA is dependent on the area dA, the differential solid angle, $d\Omega$, which in spherical coordinates is equal to

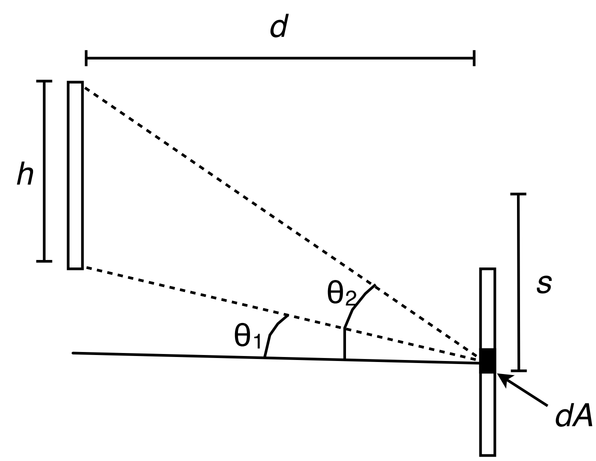

Fig. 1: Vertical Schematic of Counter Configuration

$$sin(\theta)d\theta d\phi$$

and the flux of particles through dA, which has a sine squared dependence on θ and additional cosine dependences on θ and ϕ. The first dependence is characteristic of the angular intensity of the cosmic muons, and the latter two are included as part of the flux's constraint that particles move in directions perpendicular to dA. Thus, the differential particle rate element is

$$dR = Fsin^2(\theta)cos(\theta)sin(\theta)cos(\phi)d\theta d\phi dA$$
$$= Fsin^3(\theta)cos(\theta)cos(\phi)d\theta d\phi dA$$

where F is the vertical muon flux, or F ≅ 88 particles $m^{-2}s^{-1}sr^{-1}$. By integrating this expression over the θ and ϕ ranges, we may determine the number of particles passing through the first detector and *dA* on the second in Figure 1. For the angles θ_1 to θ_2 in the integration limits, we use

$$\theta_1 = arctan(\frac{s - h/2}{d})$$
$$\theta_2 = arctan(\frac{s + h/2}{d})$$

with similar equations for ϕ_1 and ϕ_2 (where *s* and *h* are replaced respectively by *x* and *w*). With these as our integration limits, we then take the integral

$$F \int_{\theta_1}^{\theta_2} sin^3(\theta)cos(\theta)d\theta \int_{\phi_1}^{\phi_2} cos(\phi)d\phi$$

The precise final step in determining the muon incidence rate would be to integrate this result over all differential area elements of the counter on the right in Figure 1, accounting for the changing angles of θ and ϕ. However, we rely on an approximation to simplify matters. Due to the large separation *d* (1.63 m) of the counters relative to their sizes, we assert that all of the specific solid angles "seen" by each differential element on the right counter are approximately the same. This enables us to calculate this last integral and simply multiply it by the area of the counter to determine the total number of particles passing through both. When our values are inserted into these equations, our result is a muon count rate of approximately R = 0.023 or 0.02 Hz passing through both counters.

Phase 2:

This phase features a muon telescope, where several parallel counters constitute one board, and another similarly constructed board is below and oriented 90° with respect to the first. Here, our objective is again to determine the cosmic muon flux through the two counters.

Because our detectors' effective areas are roughly two meters wide by three meters in length and are thus not very long and thin, we first make the approximation that we can roughly model these counters by circular ones of the same area. Figure 2 displays our approximated configuration. The solid angle "seen" by a differential area *dA* is again calculated, and this time we take the point in the center of the bottom counter to be *dA* so that circular symmetry can be employed. To find the particle incidence rate through this differential point, the equation that we must integrate is this:

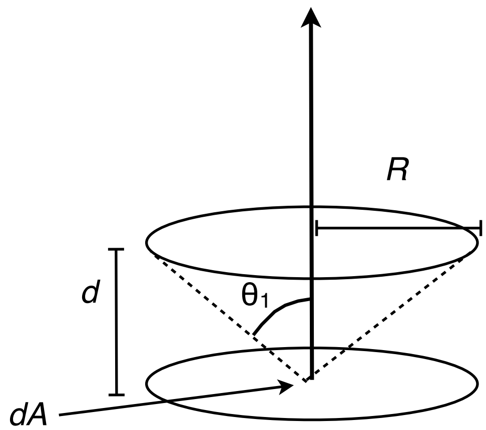

Figure 2: Approximation Used in Phase 2

$$F \int_0^{2\pi} d\phi \int_0^{\theta_1} cos^2(\theta) cos(\theta) sin(\theta) d\theta$$

where F is again the vertical flux of muons, the cosine squared term reflects the angular dependence of this flux, the next cosine term projects the flux onto the axis through the circular counters, and the sine term is part of the solid angle definition.

To determine the angle θ_1, we noted that the tangent of θ_1 is equal to *R*/*d*, so

$$\theta_1 = arctan(\frac{R}{d})$$

Given that *R* in our case would be 1.36 m and our first of four values for *d* is set at .25 m, θ_1 is 1.39 radians. Carrying out the integral above, we find a rate through *dA* of 138 Hz.

Although Phase 1's method of finding the total flux through the counters by multiplying the flux through *dA* by the total counter area does not work very well when considering small counter separation distances, we nevertheless applied this process in each case to receive an upper limit to the total particle incidence rate. The second column in Table 1 displays the results for various values of *d*.

With separations of .25, .5, 1, and 2 meters, the approximation that the counters are sufficiently separated so that each *dA* sees essentially the same solid angle is not very appropriate. The counters are much too close together for the values determined to be accepted as precise. They are only upper limits to the particle incidence rate. For our approximation, we assumed all differential area elements *dA* can be modeled by the one in the center of the circle. For different differential area elements, the average angle θ seen by the element would be larger, and therefore the cosine squared term in the integral would cause any given differential element of solid angle to contribute less on average to the total flux across *dA*. Thus, our approximation is high in all cases.

After this approximation, we made a more precise estimate of the muon incidence rate through the two counters. For this process, refer to Figures 3 & 4. We first

Separation Distance d (m)	Approximated Muon Incidence Rate (Hz)	Precise Muon Incidence Rate (Hz)
0.25	810	723
0.5	798	645
1	710	507
2	431	312

Table 1: Phase 2 Results

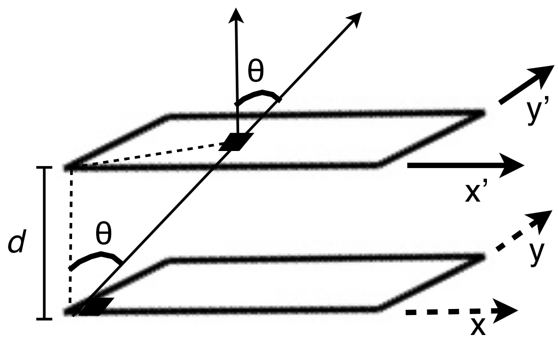

Figure 3: Precise Estimate of Muon Incidence Rate for LArIAT Phase 2.

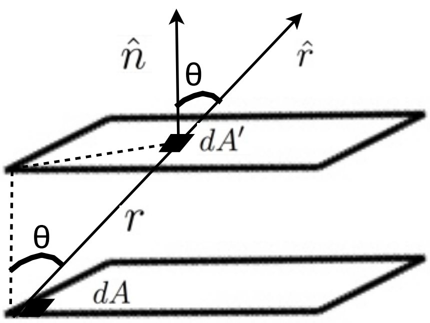

Figure 4: Additional Vectors and Calculation Aides of Phase 2.

consider the muon incidence rate through a differential area element *dA* in the corner of the lower counter and another differential area element *dA'* at any point in the upper counter. The angle θ between the vertical and the line connecting *dA* and *dA'* is equal to

$$arccos(\frac{d}{(x'-x)^2+(y'-y)^2+d^2})$$

where x' is the x coordinate of *dA'* on the top counter and x is the x coordinate of *dA* on the bottom counter. The same applies to y' and y, and d is the distance between the counters. The muon incidence rate through these two area elements will have a cosine squared dependence due to the angular intensity of muon flux, will have another cosine dependence due to the fact that flux is perpendicular to the plane through which it is calculated, and must also take into account both the area of *dA* and the solid angle it sees. The differential solid angle defined in Cartesian coordinates is

$$\frac{\hat{r}\cdot\hat{n}}{r^2}dx'dy'$$

These vectors and lengths are shown in Figure 4. For a given *dA*, we need to integrate over all *dA'*, and the final step is to integrate over all *dA*. Thus, the pertinent integral is

$$\int_{y_1}^{y_2}\int_{x_1}^{x_2}\int_{y'_1}^{y'_2}\int_{x'_1}^{x'_2} cos^3(\theta)\frac{\hat{r}\cdot\hat{n}}{r^2}dx'dy'dxdy$$

Since both vectors in the dot product are of unit magnitude, this becomes

$$\int_{y_1}^{y_2}\int_{x_1}^{x_2}\int_{y'_1}^{y'_2}\int_{x'_1}^{x'_2} cos^3(\theta)\frac{cos(\theta)}{r^2}dx'dy'dxdy$$

Because r is just the distance between the two counters in the direction of the vector between *dA* and *dA'* and because θ has already been defined above, the integral thus becomes

$$\int_{y_1}^{y_2}\int_{x_1}^{x_2}\int_{y_1'}^{y_2'}\int_{x_1'}^{x_2'} cos^4(arccos(\frac{d}{\sqrt{(x'-x)^2+(y'-y)^2+d^2}}))\frac{1}{(x'-x)^2+(y'-y)^2+d^2}dx'dy'dxdy$$

$$=\int_{y_1}^{y_2}\int_{x_1}^{x_2}\int_{y_1'}^{y_2'}\int_{x_1'}^{x_2'} \frac{d^4}{((x'-x)^2+(y'-y)^2+d^2)^3}dx'dy'dxdy$$

Due to the complexity of this integral, we used numerical methods to perform this calculation, with the x, y, x', and y' bounds being the lengths and widths of the two identical counters. For counter lengths (x'-x) = 1.82m and widths (y'-y) = 3.2 m, our results are in the third column of Table 1. For each separation distance, this method yielded results that were lower than the initial approximation. This change is expected, since the approximation can only be made by considering the point on the bottom counter that grants the smallest θ angles, and therefore the largest muon incidence rates.

To test our precise method for agreement with the first approximation at separation distances where the approximation models the situation well, we compared the results of the two with d = 50 m. We found that the two differed little. The precise method yielded a muon incidence rate of about 1.20 Hz, and the first approximation yielded a rate of 1.21Hz. This displays an error of only about 1%, which supports the idea that the more precise method is in fact viable.